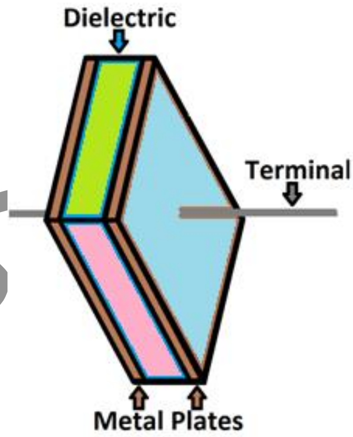
Capacitor and Capacitance

1. Capacitor and its Classification: Concept of Capacitance of a Capacitor and its Unit:

Capacitor is that electrical equipment which can store certain amount of charge at a certain potential drop across it. It is normally constructed by two conducting foils of certain geometric shape when separated by air or any other dielectric medium. Thus capacitor has two types which are dielectric capacitor and air capacitor. Again on the basis of geometric shape of the metallic foils used the capacitor has also three types which are parallel plate capacitor, spherical capacitor and cylindrical capacitor.



The basic principle by which the charge will be stored in a capacitor is a fact that, when a plate of the capacitor is charged by some amounts of positive charge then the other plate will become charged by equal amount of induced negative charge either by electrostatic induction or by dielectric polarization. So finally a potential drop will appear across the two plates of that capacitor due to the storage of charge in that capacitor.

Capacitor Symbol

So if $\mathbf{Q}$ be the amount of charge which is stored within the capacitor and for such storage, the potential drop appeared across it for corresponding work done of charging is $\mathbf{V}$. In that case $\mathbf{Q} \propto \mathbf{V}$ and $\mathbf{Q} = \mathbf{CV}$. Here $\mathbf{C}$ is proportionality constant and it is called Capacitance of the capacitor.

Since in this case, for $\mathbf{V} = 1$, $\mathbf{Q} = \mathbf{C}$, the capacitance of a capacitor is defined by that amount of charge which is required to store within the capacitor to appear unit potential drop across it. Since $\mathbf{C} = \frac{\mathbf{Q}}{\mathbf{V}}$, so SI unit of capacitor is Coulomb/ Volt = Farad where its cgs unit is Stat Coulomb / esu of potential = Stat Farad or esu of capacitance and the conversion is

$$\mathbf{1\ Farad} = \frac{\mathbf{1\ Coulomb}}{\mathbf{1\ Volt}} = \frac{\mathbf{3 \times 10^9\ esu\ of\ charge}}{\frac{\mathbf{1}}{\mathbf{300}}\ \mathbf{esu\ of\ potential}} = \mathbf{9 \times 10^{11}\ esu\ of\ Capacitance}$$

2. Energy Stored for Charging of a Capacitor:

The electrostatic potential energy stored in a capacitor will be equal to the total work done required for charging. So if during intermediate process of charging, for the storage of q amount charge in a capacitor, dV amount of potential drop will appear across it then the total energy stored for complete charging will be

$$U = W = \int_0^V q.dV = \int_0^V CV.dV = \frac{1}{2}CV^2 = \frac{1}{2}.QV = \frac{Q^2}{2C}$$

3. Energy Loss for the Connection of two Charged Capacitors by a Conducting Wire:

We now consider two capacitors in which respectively Q_1 and Q_2 amount of charge are stored. Thus the potential appeared in this two capacitors are V_1 and V_2 respectively. Hence initially the energy stored in this two capacitor together is $U_1 = \frac{1}{2}.[C_1V_1^2 + C_2V_2^2]$

Now if this two charged capacitors be connected by a conducting wire the charge exchange between that two capacitors will occur up till the potential drop of both the capacitor will be the same. If this same potential drop be V then for their finally stored charge Q_1^0 and Q_2^0 respectively, the total final energy stored will be

$$U_2 = \frac{1}{2}[Q_1^0V + Q_2^0V] = \frac{1}{2}[C_1.V.V + C_2.V.V] = \frac{1}{2}[C_1 + C_2]V^2$$

Where by charge conservation, we have $Q_1 + Q_2 = Q_1^0 + Q_2^0$

And $C_1V_1 + C_2V_2 = C_1V + C_2V$ Or, $V = \frac{C_1V_1+C_2V_2}{C_1+C_2} = \frac{Q_1+Q_2}{C_1+C_2}$ Thus we see that

$$U_1 - U_2 = \frac{1}{2}.[\{C_1V_1^2 + C_2V_2^2\} - (C_1 + C_2)V^2] = \frac{1}{2}.\left[\{C_1V_1^2 + C_2V_2^2\} - (C_1 + C_2)\left(\frac{C_1V_1+C_2V_2}{C_1+C_2}\right)^2\right]$$

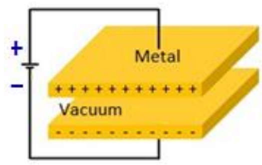
$= \frac{1}{2}\frac{C_1C_2}{C_1+C_2}.(V_1 - V_2)^2 > 0$ i.e. $U_1 > U_2$ and energy loss will occur. This energy loss will be $\Delta U = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}.(V_1 - V_2)^2$

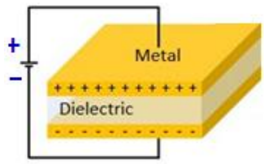
The cause of such energy loss is Julian heating effect for the passage of charge or current flow through that connecting wire. Hence the heat produced in this process will be

$$Q = \frac{W}{J} = \frac{\Delta U}{J} = \frac{1}{2J}\left[\frac{C_1C_2}{C_1 + C_2}\right].(V_1 - V_2)^2$$

4. Determination of Capacitance of Several Capacitors:

a) Parallel Plate Capacitor:



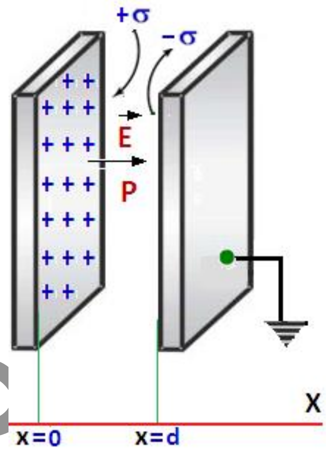


For such capacitor two parallel metallic plates, each having cross section A are separated either by air or by dielectric with comparatively separation d. Thus for parallel plate air capacitor, if one plate be charged by positive charge $+Q$ then by electrostatic induction the equal but opposite charge $-Q$ will be induced in the other plate.

Here as shown in figure if that other plate be made earthed at zero potential then the effective electric field at any intermediate point **P** between that two plates will be $\mathbf{E = \frac{\sigma}{2\epsilon_o} + \frac{\sigma}{2\epsilon_o} = \frac{\sigma}{\epsilon_o}}$

Since $\boldsymbol{\sigma}$ = surface density of charge $= \frac{Q}{A}$ and $\mathbf{E = -\frac{dV}{dx}}$, we should have $\int dV = -\int E.dx$ by which we get

$$\int_V^0 dV = -\int \frac{\sigma}{\epsilon_o}.dx = -\frac{1}{\epsilon_o}\frac{Q}{A}\int_0^d dx \text{ Or, } V = \frac{Qd}{A\epsilon_o}$$

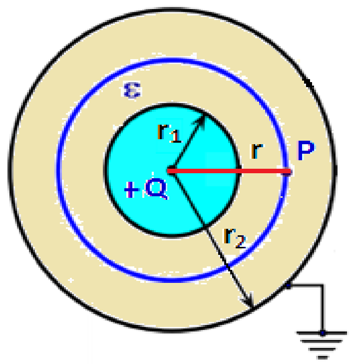


Then we get the capacitance of parallel plate air capacitor in **SI** system $C = \frac{Q}{V} = \frac{A\epsilon_o}{d}$ similarly the capacitance of a parallel plate air capacitor in cgs system will be $C = \frac{Q}{V} = \frac{A}{4\pi d}$.

Again for parallel plate dielectric capacitor, its capacitance will similarly become $C = \frac{Q}{V} = \frac{A\epsilon_o k}{d}$ [SI] and $C = \frac{Q}{V} = \frac{kA}{4\pi d}$ [cgs]

b) Spherical Capacitor:

To construct such capacitor we take two concentric spherical conducting shells separated either by air or dielectric. So if the outer shell be made grounded with inner shell charged by the charge **Q** then for their inner and outer radius r_1 and r_2 , the electrostatic field at any intermediate point **P** within two shells of that spherical air capacitor at a distance **r** from the common center of that capacitor will be



$$E = -\frac{dV}{dr} = \frac{1}{4\pi\epsilon_o}.\frac{Q}{r^2} \text{ Or,} -\int_V^0 dV = \frac{1}{4\pi\epsilon_o}.\int_{r_1}^{r_2}\frac{Q}{r^2}dr$$

$$\text{Or, } V = \frac{Q}{4\pi\epsilon_o}.\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

And finally the capacitance of a spherical air capacitor in **SI** system will be $C = \frac{Q}{V} = \frac{4\pi\epsilon_o r_1 r_2}{r_2 - r_1}$.

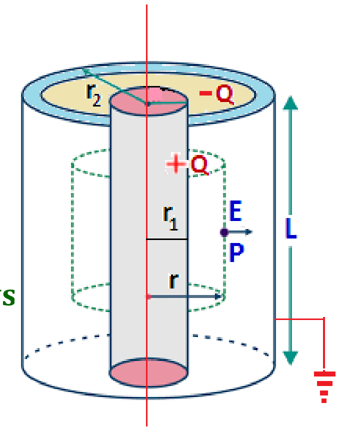
Similarly, this capacitance of spherical air capacitor in cgs system will be $C = \frac{Q}{V} = \frac{r_1 r_2}{r_2 - r_1}$.

Again for spherical dielectric capacitor, its capacitance will similarly become

$$C = \frac{Q}{V} = \frac{4\pi\epsilon_o k r_1 r_2}{r_2 - r_1}[SI] \quad \text{and} \quad C = \frac{Q}{V} = \frac{k r_1 r_2}{r_2 - r_1}[cgs]$$

c) Cylindrical Capacitor:

To construct such capacitor we take two concentric cylindrical conducting shells separated either by air or dielectric. So if the outer shell be made grounded with inner shell charged by the charge **Q** then for their inner and outer radius r_1 and r_2, the electrostatic field at any intermediate point **P** within two shells of that cylindrical air capacitor at a distance r from the common axis of that capacitor will be



$$E = -\frac{dV}{dr} = \frac{\sigma}{\epsilon_o} = \frac{1}{\epsilon_o}.\frac{Q}{2\pi rl}$$

$$\text{Or, } -\int_V^0 dV = \frac{Q}{2\pi\epsilon_o l}.\int_{r_1}^{r_2}\frac{1}{r}\,dr \quad \text{Or, } \quad V = \frac{Q}{2\pi\epsilon_o l}.\ln\left(\frac{r_2}{r_1}\right)$$

and finally the capacitance of a cylindrical air capacitor in SI sys

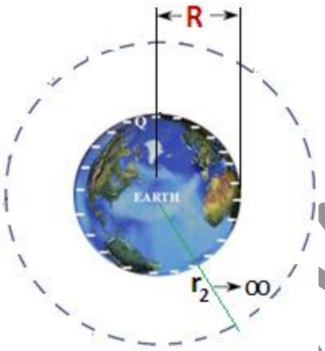
$$C = \frac{Q}{V} = \frac{2\pi\epsilon_o l}{\ln\left(\frac{r_2}{r_1}\right)}.$$

Similarly, this capacitance of cylindrical air capacitor in cgs system will be $C = \frac{Q}{V} = \frac{l}{2\ln\left(\frac{r_2}{r_1}\right)}$. Again for cylindrical dielectric capacitor, its capacitance will be

$$C = \frac{Q}{V} = \frac{2\pi\epsilon_o kl}{\ln\left(\frac{r_2}{r_1}\right)}\ [\text{SI}] \quad \text{and} \quad C = \frac{Q}{V} = \frac{kl}{2\ln\left(\frac{r_2}{r_1}\right)}\ [\text{cgs}]$$

5. Capacitance of Earth or any Single Spherical Body:

Here we consider a sphere of radius r. This can now be treated as a spherical air capacitor with $r_1 = r$ and $r_2 \to \infty$. So the capacitance of a spherical body in SI system will become



$$C = \lim_{r_2\to\infty}\frac{4\pi\epsilon_o r_1 r_2}{r_2 - r_1} = \lim_{r_2\to\infty}\frac{4\pi\epsilon_o r_1}{1-\frac{r_1}{r_2}} = 4\pi\epsilon_o r_1 = 4\pi\epsilon_o r$$

And thus, this capacitance of a single sphere in cgs system will be $C = r = \text{the radius of that sphere}$. Hence by taking earth to be a sphere of radius R = 6400 km, the capacitance of earth should be

$$C = 4\pi\epsilon_o r = 4\pi\epsilon_o R = \frac{1}{\left(\frac{1}{4\pi\epsilon_o}\right)}R = \frac{1}{9\times10^9}\times\left(6400\ \times 10^3\right)\text{Farad}$$

$$= \frac{1}{9\times10^9}\times\left(6400\ \times10^3\right)\times10^6\ \mu F$$

Finally the capacitance of spherical earth is $\left.C\right]_{earth} = 711.11\ \mu F$

Solved Problems

1. In a parallel plate capacitor with air between its plates, each plate has an area of $6\times10^{-3}m^2$ and distance between the plates is 3 mm. (i) calculate the capacitance of the capacitor. (ii) If this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor? (ii)

What will the charge on the capacitor if a 3 mm thick mica sheet (of dielectric constant K = 6) is inserted between the plates?

Ans: Given: $A = 6 \times 10^{-3} m^2, d = 3 \times 10^{-3}$ m, $V = 100$ Volt

(i) Capacitance $C = \frac{A\epsilon_0}{d} = \frac{6\times10^{-3}\times8.85\times10^{-12}}{3\times10^{-3}} = 17.7 \times 10^{-12}$ F $= 18$ pF

(ii) Charge on the capacitor $q = CV = 18 \times 10^{-12} \times 100 = 1.8 \times 10^{-9}$C. The charges on the plates are respectively $+1.8 \times 10^{-9}$C and -1.8×10^{-9}C

(iii) Capacitance of the capacitor with the dielectric sheet $C' = K\frac{A\varepsilon_0}{d} = KC_0 = 6 \times 18 = 108$ pF. Charge on the capacitor, $q' = C'V = 108 \times 10^{-12} \times 100 = 10.8 \times 10^{-9}$C

2. What is the area of the plate of a 2F parallel plate capacitor with plate separation of 0.5 cm? Why do ordinary capacitors have capacitance of the order of microfarads?

Ans: Given: $C = 2F, d = 0.5 \times 10^{-2}$ m $C = \frac{A\varepsilon_0}{d}$

Thus we get $A = \frac{Cd}{\varepsilon_0} = \frac{2\times0.5\times10^{-2}}{8.85\times10^{-12}} = 1130 \times 10^6 m^2 = 1130\ km^2$.

Since, this is a very large and impractical size; the ordinary capacitors of reasonable size have capacitance of the order of microfarads.

3. How and by what % will the capacitance of a capacitor be affected when the (i) area of its plates is increased by 50% and plate separation is decreased by 25% (ii) area of the plates is halved and the plate separation is made one third?

Ans: (i) Initial capacitance $C = \frac{A\epsilon_0}{d}$. New area of the plates $A_2 = A + 50\%$ of $A = \frac{3}{2}A$. New separation $d_2 = d - 25\%$ of $d = \frac{3}{4}d$, New capacitance $C_2 = \frac{A_2\varepsilon_0}{d_2} = \frac{3A\varepsilon_0\times4}{2\times3d} = 2.\frac{A\varepsilon_0}{d} = 2C$

Percentage increase in capacitance $= \frac{2C-C}{C} \times 100 = 100\%$

(ii) New area $A_2 = \frac{A}{2}$, New separation $d_2 = \frac{d}{3}$ Hence capacitance $C_2 = \frac{A_2\varepsilon_0}{d_2} = \frac{3A\varepsilon_0}{2d} = \frac{3}{2}C$

Increase in capacitance $\Delta C = C_2 - C = \frac{C}{2}$ Thus percentage increase $= \frac{\Delta C}{C} = 50\%$

4. Find the ratio of the potential difference that must be applied across the parallel and series combination of two capacitors C_1 and C_2 with their capacitance in the ratio of 1:2 so that the energy stored in the two cases becomes the same.

Ans: Given: $\frac{C_1}{C_2} = \frac{1}{2}$ and $C_2 = 2C_1 C_P = C_1 + C_2 = C_1 + 2C_1 = 3C_1 C_P = \frac{C_1C_2}{C_1+C_2} = \frac{C_1\times2C_1}{C_1+2C_1} = \frac{2C_1}{3}$

Let V_P and V_S be the p.d. applied across the parallel and the series combination. Here $\frac{1}{2}C_PV_P^2 = \frac{1}{2}C_SV_S^2$ Thus $\frac{V_P^2}{V_S^2} = \frac{C_S}{C_P} = \frac{2C_1}{3\times3C_1} = \frac{2}{9}$ and $\frac{V_P}{V_S} = \frac{\sqrt{2}}{3}$

5. Two capacitors of unknown capacitances are connected (i) in series and (ii) in parallel. If the net capacitance in the two combinations is6 μF and 25 μF respectively, find their capacitances.

Ans: Given: $C_s = 6\ \mu F$ and $C_P = 25\ \mu F$

(i) In series $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$ or $\frac{1}{6} = \frac{C_1+C_2}{C_1C_2}$ or $C_1 + C_2 = \frac{C_1C_2}{6}$ .... (1) .

Also $C_P = C_1 + C_2$ or $C_1 + C_2 = 25$ .... (2). Using Eq. (1) and Eq. (2), we get, $C_1C_2 = 150$

Now$(C_1 - C_2)^2 = (C_1 + C_2)^2 - 4C_1C_2 = (25)^2 - 4\times150 = 625 - 600 = 25$

Thus $C_1 - C_2 = 5$ ...... (3). Solving Eqs. (2) and (3),we get, $C_1 = 15\ \mu F$ and $C_2 = 10\ \mu F$

6. Three capacitors of capacitance 2 pF, 3 pF and 4 pF are connected in parallel. (i) What is the total capacitance of the combination? (ii) Determine the charge on each capacitor if the combination is connected to 100 V supply.

Ans: Given: $C_1 = 2\times10^{-12}F, C_2 = 3\times10^{-12}F$ and $C_3 = 4\times10^{-12}F$

(a) In parallel, $C_P = C_1 + C_2 + C_3 = 9\times10^{-12}F$

(b) In parallel, V remains the same for all capacitors

Thus $q_1 = C_1V = 2\times10^{-12}\times100 = 2\times10^{-10}C$ $q_2 = C_2V = 3\times10^{-12}\times100 = 3\times10^{-10}C$ and $q_3 = C_3V = 4\times10^{-12}\times100 = 4\times10^{-10}C$

7. Three capacitors of capacitance 9 mF each are connected in series. (a) What is the total capacitance of the combination? (b) What is the potential difference across each capacitor if the combination is connected to a 120 V supply?

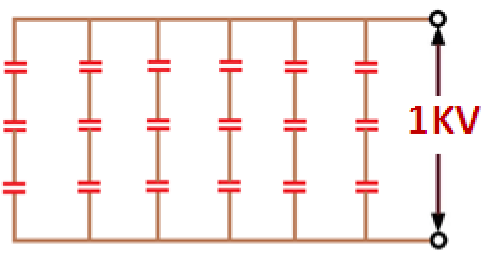
Ans: (a) Let their total capacitance be C_S.

$$\text{Thus } \frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{1}{3} \text{ Or, } C_S = 3\ \mu F$$

(b) Charge on each capacitor, $q = C_SV = 3\times10^{-6}\times120 = 3.6\times10^{-4}C$

Potential difference across each capacitor $V = \frac{q}{C} = \frac{3.6\times10^{-4}}{9\times10^{-6}} = 40\ V$

8. An electrical technician requires a capacitance of 2 μF in a circuit across a potential difference of 1 kV. A large number of 1 μF capacitors are available to him; each can withstand a potential difference of not more than 400 V.



Suggest a possible combination that requires a minimum number of capacitors.

Ans: Let n capacitors each of $1\ \mu F$ be connected to withstand a potential difference of 1000 V. Thus potential difference across one capacitor $V = \frac{1000}{n}$

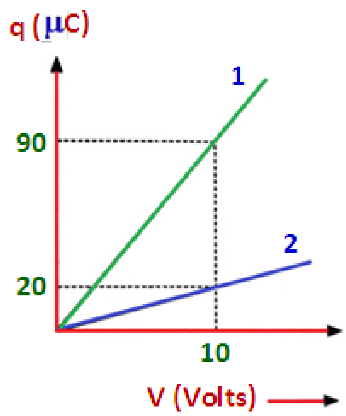
Hence $\frac{1000}{n} = 400$ and then $n = 2.5$. As the number of capacitors cannot be in fraction the number of capacitor is $n = 3$.

Thus net capacitance of three $1\ \mu F$ capacitors connected in series

$\frac{1}{C_1} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1}$ i.e. $C_1 = \frac{1}{3}\mu F$. Since the net capacitance required is $2\ \mu F$, the technician will have to connect a number (m) of each series combinations in parallel.

Hence $m \times \frac{1}{3} = 2$ i.e. $m = 6$ And minimum number of capacitors used $= 6 \times 3 = 18$

9. The charge 'q' versus potential difference 'V' graphs for the series and the parallel combination of two capacitors are shown in the figure. (i) What does the slope of a line represent? (ii) Identify the lines representing the two combinations and (iii) Find the capacitances of the two capacitors.



Ans: (i) We know, $C = \frac{q}{V}$ Slope $\left(\frac{\Delta q}{\Delta V}\right)$ of the line represents the net capacitance. (ii) Since the slope of line (1) is greater than that of line (2).

Hence Line (1) represents the parallel combination and line (2) represents the series combination. (iii) Using line (1), $C_P = \frac{q}{V} = \frac{9\times10^{-6}}{10} = 9 \times 10^{-6}\ F = 9\ \mu F$. Let C_1 and C_2 be the capacitances of the two capacitors

Thus $C_1 + C_2 = 9$ …… (1) Similarly, from line (2), $C_s = \frac{C_1C_2}{C_1 + C_2} = \frac{20\times10^{-6}}{10} = 2\ \mu F$

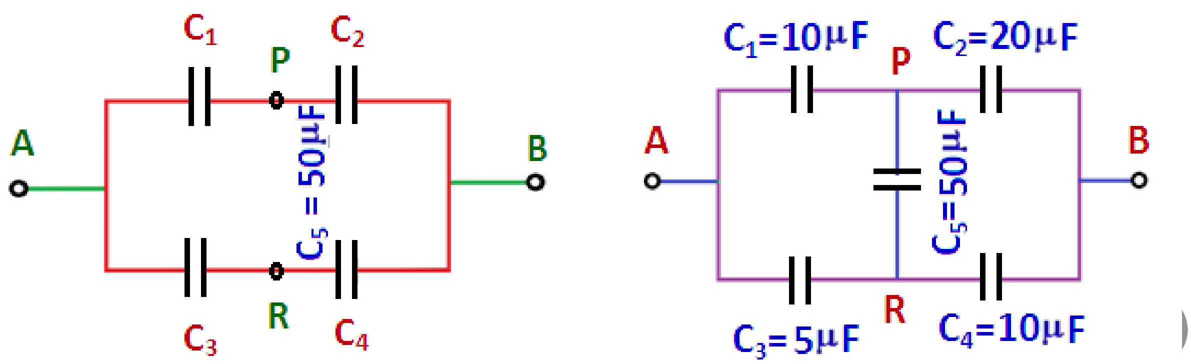
Or $C_1C_2 = 2(C_1 + C_2) = 18$ … …. (2)

Solving Eqns. (1) and (2), we get, $C_1 = 3\ \mu F$ and $C_2 = 6\ \mu F$

10. Three identical capacitors each of capacitance $3\ \mu F$ are connected, in turn, in series and in parallel combination to the common source of V volts. Find the ratio of the energies stored in the two combinations.

Ans: Net capacitance in series $\frac{1}{C_S} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1 \therefore C_S = 1\ \mu F$. In parallel combination $C_P = 3 + 3 + 3 = 9\ \mu F$ Since $W = \frac{1}{2}CV^2 \therefore \frac{W_S}{W_P} = \frac{\frac{1}{2}C_SV^2}{\frac{1}{2}C_PV^2} = \frac{C_S}{C_P} = \frac{1}{9}$ This is the ratio of energy stored.

11. Calculate the equivalent capacitance between points A and B in the circuit shown. If a battery of 10 V is connected across A and B, calculate the charge drawn from the battery.



Ans: In the circuit APBRA the condition of balance of wheat stone bridge is satisfied. So points P and Q are at the same potential.

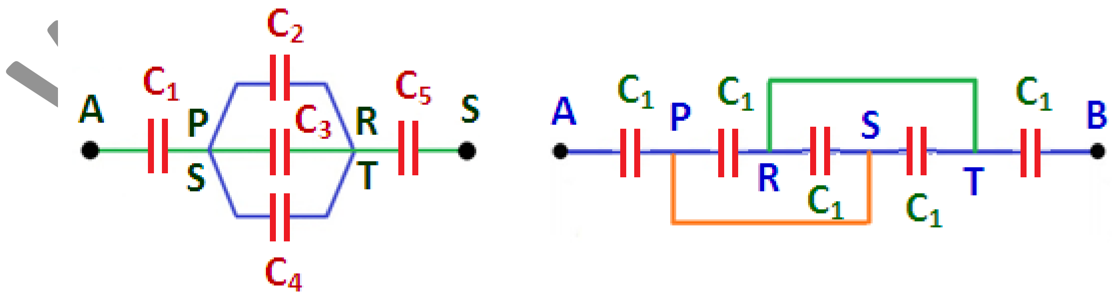
Hence, $50\ \mu F$ capacitor is practically of no consequence. The circuit gets reduced as shown in the figure.

Net capacitance of armAPB, $C' = \frac{C_1 C_2}{C_1 + C_2} = \frac{10\times20}{10+20} = \frac{20}{3}\ \mu.$

Net capacitance of arm ARB is $C' = \frac{5\times10}{15} = \frac{10}{3}\ \mu F$. Hence equivalent capacitance between A and B is $C = C' + C'' = \frac{20}{3} + \frac{10}{3} = 10\ \mu F$

So the charge drawn, $q = CV = 10 \times 10 = 100\ \mu C$

12. (i) Find the equivalent capacitance between A and B in the combination given below. Each capacitor is of $2\ \mu F$ capacitance. (ii) If a dc source of 7 V is connected across AB, how much charge is drawn from the source and what is the energy stored in the network?



Ans: (i) The equivalent circuit Net capacitance between P and R,

$$C = C_2 + C_3 + C_4 = 2 + 2 + 2 = 6\ \mu F.$$

For net capacitance between A and B $\frac{1}{C_{net}} = \frac{1}{2} + \frac{1}{6} + \frac{1}{2} = \frac{7}{6}$ i.e. $C_{net} = \frac{6}{7}\ \mu F$

(ii) Since $q = CV$, charge drawn from the source, $= C_{net}V = \frac{6}{7} \times 7 = 6\ \mu C$. Energy stored $= \frac{1}{2}CV^2 = \frac{1}{2} \times \frac{6}{7} \times 7^2 = 21\ \mu J$

13. The plates of a parallel plate capacitor have an area of 90 cm^2 each and are separated by 2.5 mm. (a) Find the capacitance of the capacitor. (b) If the capacitor is charged by connecting it to a 400 V supply, how much energy is stored by the capacitor? (c) Calculate the energy stored per unit volume of the capacitor.

Ans: Given: $A = 90 \times 10^{-4} m^2, d = 2.5 \times 10^{-3}\ m$

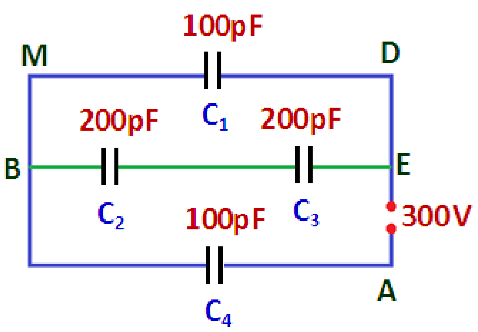
(a) $C = \frac{A\epsilon_0}{d} = \frac{90 \times 10^{-4} \times 8.85 \times 10^{-12}}{2.5 \times 10^{-3}} = 31.86 \times 10^{-12} F$

(b) Energy stored in the capacitor $U = \frac{1}{2}CV^2 = \frac{1}{2} \times 31.86 \times 10^{-12} \times 16 \times 10^4 = 2.55 \times 10^{-6} J$

(c) Energy stored per unit volume $u = \frac{U}{Ad} = \frac{2.55 \times 10^{-6}}{9 \times 10^{-3} \times 2.5 \times 10^{-3}} = 0.113\ Jm^{-3}$

14. Obtain the equivalent capacitance of the network shown in the figure. For a 300 V supply, determine the charge and the voltage across each capacitor.

Ans: Net capacitance of C_2 and C_3 betweenB and E, i.e., C_{23} is given by



$$\frac{1}{C_{23}} = \frac{1}{200} + \frac{1}{200} \text{ or, } C_{23} = 100\ pF .$$

As capacitor C_1 in the arm BMDE and C_{23} in arm BE are connected in parallel, net capacitance of C_1 and C_{23} is $C_{123} = C_1 + C_{23} = 100 + 100 = 200\ \mu F$

Now for the net capacitance of the network across the supply

$$\frac{1}{C} = \frac{1}{C_{123}} + \frac{1}{C_4} = \frac{1}{200} + \frac{1}{100} \text{ or, } C = \frac{200}{3}\ \mu F$$

Thus charge supplied by the battery

$q = CV = \frac{200}{3} \times 300\ pC = 200 \times 100 \times 10^{-12} = 2 \times 10^{-8} C$ and charge on C_4 = charge on $C_{123} = 2 \times 10^{-8} C$ i.e. $q_4 = 2 \times 10^{-8} C$ and $V_4 = \frac{2 \times 10^{-8}}{100 \times 10^{-12}} = 200\ V.$

So the potential difference between points B and E $= 300 - 200 = 100\ V$.

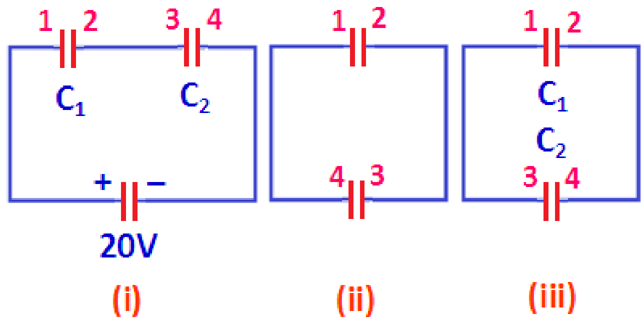
Thus V_1 (Potential difference across C_1)

$$= 100\text{ V and } q_1 = C_1V_1 = 100 \times 10^{-12} \times 100 = 10^{-8}\text{ C}$$

Now $V_2 + V_3 = 100$. Thus $V_2 = V_3 = 50$ Volt and

$$q_3 = q_2 = C_2V_2 = 200 \times 10^{-12} \times 50 = 10^{-8}\text{ C}$$

15. Two capacitors C_1 and C_2 of capacitances 6 μF and 3 μFare connected across battery of 20 V, as shown in Fig.(i). The battery is disconnected and the charged capacitors are reconnected as in Figs. (ii) and (iii). Find the final charge on each capacitor in the two cases (ii) and (iii). Here (1, 2) and (3, 4) are the plates of C_1 and C_2 respectively.



Ans: Initially in Fig. (i), capacitors were connected in series. So they had equal charges. Initial charge on each capacitor,

$$q = C_SV = \frac{C_1C_2}{C_1 + C_2}V = \frac{6\times3}{9} \times 20 = 40\ \mu C$$

(i) When the charged capacitors are reconnected, as in Fig.

(ii) They lose their charges and become neutral. Hence final charge on each = 0

(ii) If the charged capacitors were connected as in Fig.

(iii) Plates with like charges get connected. They share the charge till they attain a common potential (V). Let q_1 and q_2 are final charges on them.

$$\text{Hence } V = \frac{q_1}{C_1} = \frac{q_2}{C_2} \therefore \frac{q_1}{q_2} = \frac{C_1}{C_2} = \frac{6}{3} = 2 \therefore q_1 = 2q_2 .....(1)$$

But $q_1 + q_2 = \text{Total charge} = 2 \times 40 = 80\ \mu C$ or $q_1 + q_2 = 80$ …… (2) . Solving Eqns. (1) and (2), we get $q_1 = \frac{160}{3}\ \mu C$ and $q_2 = \frac{80}{3}\ \mu C$