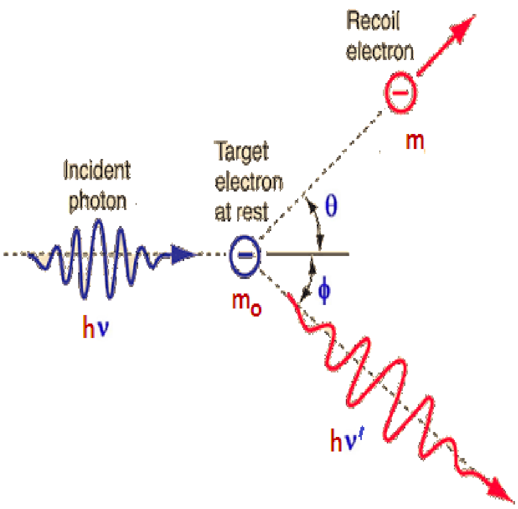
Compton's Effect or Compton's Scattering:

If gamma photon of intermediate energy hits an electron, initially at rest then part of energy of that photon will be absorbed by that electron for which the electron will be recoiled with some momentum and that incoming photon with remaining energy will be scattered from that electron. This is Compton's scattering and by this effect which ensures the particle aspect of radiation, the wave length shifting of the scattered to incoming radiation will appear.



As shown in figure we see that the incoming photon of energy $h\nu$ and momentum $\frac{h\nu}{c}$ incidence on an electron having rest mass m_o which is initially at rest with rest energy m_oc^2. After scattering at an angle φ, the scattered photon has energy $h\nu'$ and momentum $\frac{h\nu'}{c}$ where the electron recoils with momentum mv (with moving mass) at an angle θ with the direction of incoming photon.

Now we have from energy conservation

$$h\nu + m_oc^2 = h\nu' + mc^2 \Rightarrow h(\nu - \nu') + m_oc^2 = mc^2 \text{ .... (1)}$$

On the other hand we have from momentum conservation

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\text{Cos}\varphi + mv\text{Cos}\theta \text{ and } \frac{h\nu'}{c}\text{Sin}\varphi - mv\text{Sin}\theta = 0 \text{ ....(2)}$$

From this equation (2) we get $\left(\frac{h\nu}{c} - \frac{h\nu'}{c}\text{Cos}\varphi\right)^2 + \left(\frac{h\nu'}{c}\text{Sin}\varphi\right)^2 = m^2v^2$ **........ (3)**

Simplifying we get $h^2(\nu^2 + \nu'^2 - 2\nu\nu'\text{Cos}\varphi) = m^2v^2c^2$ **.................... (4)**

Also we have from equation (1)

$$[h(\nu - \nu') + m_oc^2]^2 = m^2c^4 \Rightarrow h^2(\nu^2 + \nu'^2 - 2\nu\nu') + 2h(\nu - \nu')m_oc^2 + m_o^2c^4 = m^2c^4$$

.... (5)

But we have for relativistic electron $E^2 = (mc^2)^2 = m^2c^4 = m^2v^2c^2 + m_o^2c^4$

Thus we have from equation (5)

$$h^2(\nu^2 + \nu'^2 - 2\nu\nu') + 2h(\nu - \nu')m_oc^2 + m_o^2c^4 = m^2c^4 = m^2v^2c^2 + m_o^2c^4 \text{ .... (6)}$$

Or $h^2(\nu^2 + \nu'^2 - 2\nu\nu') + 2h(\nu - \nu')m_o c^2 = m^2 v^2 c^2$ .... (7)

Finally we have from equation (7) and (4)

$$h^2(\nu^2 + \nu'^2 - 2\nu\nu') + 2h(\nu - \nu')m_o c^2 = h^2(\nu^2 + \nu'^2 - 2\nu\nu' \text{Cos}\varphi)$$

Or, $(\nu - \nu')m_o c^2 = h\nu\nu'(1 - \text{Cos}\varphi) \Rightarrow \left(\frac{c}{\nu'} - \frac{c}{\nu}\right) = \frac{h}{m_o c}(1 - \text{Cos}\varphi)$

And we get the wave length shift in Compton's scattering $\Delta\lambda = \lambda' - \lambda = \lambda_c(1 - \text{Cos}\varphi)$

where λ_c is Compton's wave length and it is given by $\lambda_c = \frac{h}{m_o c}$