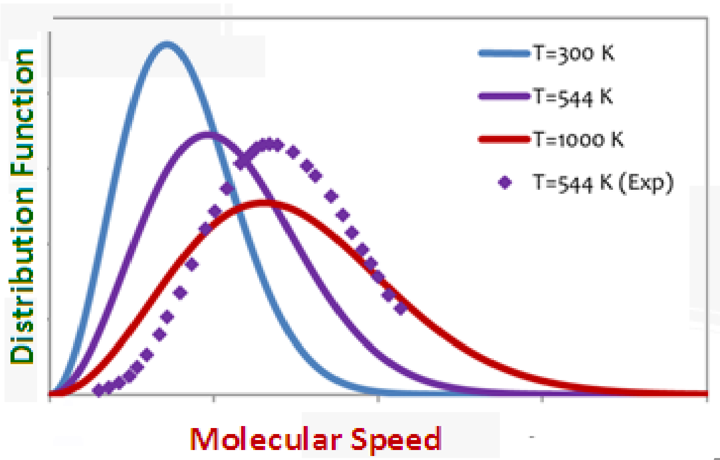
Kinetic Theory of Gas

1. Maxwell's Law of Velocity Distribution and Velocity Distribution Equation:

According to Maxwell's law, for Brownian motion of gas molecules within a close container, although different molecules has different velocity but a number of molecule may have same velocity and maximum number of molecules will possess a certain velocity, called most probable velocity. This is Maxwell's velocity distribution law.



Here by using probabilistic concept, Maxwell showed mathematically that if dn_c be the number of molecules having velocity between c and $c + dc$ then $dn_c = 4\pi n a^3 e^{-bc^2} c^2 dc$ where $n =$ no of molecule per unit volume, a and b are Maxwell's constant and also it can be shown mathematically that $a = \sqrt{\frac{b}{\pi}} = \sqrt{\frac{m}{2\pi kT}}$ and $b = \frac{m}{2kT}$ where symbols has their usual meanings.

2. Average and RMS Velocity of Gas Molecules:

Now we consider that for a given gaseous system, each of n_1 number of molecules has velocity c_1, each of n_2 number of molecules has velocity c_2, each of n_3 number of molecules has velocity c_3 ,……etc. Thus the average velocity of the gas molecule will be

$$\bar{c} = \frac{n_1c_1+n_2c_2+n_3c_3+\cdots.....}{n} = \frac{\lim_{r\to\infty}\sum_{p=0}^{r}[n_pc_p]}{n} = \frac{1}{n}\int_{c=0}^{\infty} c\, dn_c\ .$$

But by applying Maxwell's velocity distribution formula we can solve this integration by using Gamma function technique and then finally we get average velocity of the gas molecule as $\bar{c} = \sqrt{\frac{8kT}{\pi m}} = \sqrt{\frac{8RT}{\pi M}}$. Similarly the rms velocity of gas molecules will be

$$c_{rms} = \sqrt{\overline{c^2}} = \sqrt{\frac{n_1c_1^2+n_2c_2^2+n_3c_3^2+\cdots....}{n}} = \sqrt{\frac{\lim_{r\to\infty}\sum_{p=0}^{r}[n_pc_p^2]}{n}} = \sqrt{\frac{1}{n}\int_{c=0}^{\infty} c^2\, dn_c}.$$

And by the same manner of solving this integration we finally get rms velocity of gas molecules as $c_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}}$.

3. Relation between RMS Velocities of Gas Molecules with Density of Gas:

Since for gas molecules its rms velocity is given by $c_{rms} = \sqrt{\frac{3RT}{M}}$ then for one mole of ideal gas we can have $c_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3PV}{M}} = \sqrt{\frac{3P}{\frac{M}{V}}} = \sqrt{\frac{3P}{\rho}} \Longrightarrow c_{rms} \propto \frac{1}{\sqrt{\rho}}$. Thus rms velocity of gas molecule during its Brownian motion is inversely proportional to the square root of density of that given gas.