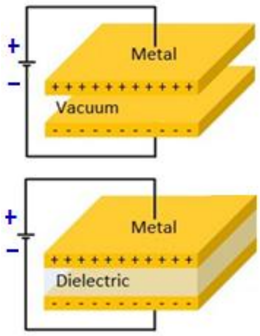
1. Capacitance of Parallel Plate, Spherical and Cylindrical Capacitors:

a) Parallel Plate Capacitor:

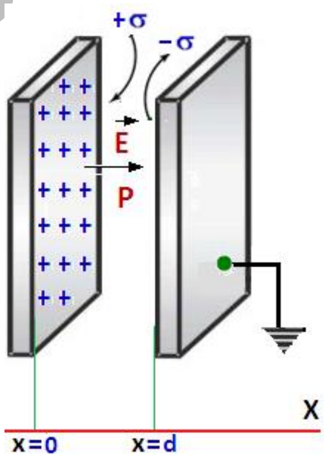
For such capacitor two parallel metallic plates, each having cross section A is separated either by air or by dielectric with comparatively separation d. Thus for parallel plate air capacitor, if one plate be charged by positive charge $+Q$ then by electrostatic induction the equal but opposite charge $-Q$ will be induced in the other plate. Here as shown in figure if that other plate be made earthed at zero potential then the effective electric field at any intermediate point P between that two plates will be



$$E = \frac{\sigma}{2\epsilon_o} + \frac{\sigma}{2\epsilon_o} = \frac{\sigma}{\epsilon_o}$$

Since $\sigma =$ surface density of charge $= \frac{Q}{A}$ and $E = -\frac{dV}{dx}$, we should have $\int dV = -\int E.dx$ by which we get

$$\int_V^0 dV = -\int \frac{\sigma}{\epsilon_o}.dx = -\frac{1}{\epsilon_o}\frac{Q}{A}\int_0^d dx \ \text{Or}, V = \frac{Qd}{A\epsilon_o}.$$



Then we get the capacitance of parallel plate air capacitor in SI system $C = \frac{Q}{V} = \frac{A\epsilon_o}{d}$. Similarly, the capacitance of a parallel plate air capacitor in cgs system will be $C = \frac{Q}{V} = \frac{A}{4\pi d}$. Again for parallel plate dielectric capacitor, its capacitance will similarly become $C = \frac{Q}{V} = \frac{A\epsilon_o k}{d}$ [SI] and $C = \frac{Q}{V} = \frac{kA}{4\pi d}$ [cgs]

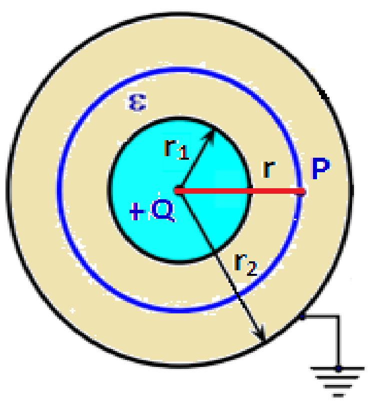
b) Spherical Capacitor:

To construct such capacitor we take two concentric spherical conducting shells separated either by air or dielectric. So if the outer shell be made grounded with inner shell charged by the charge Q then for their inner and outer radius r_1 and r_2, the electrostatic field at any intermediate point P within two shells of that spherical air capacitor at a distance r from the common center of that capacitor will be

$$E = -\frac{dV}{dr} = \frac{1}{4\pi\epsilon_o}.\frac{Q}{r^2} \ \text{Or}, -\int_V^0 dV = \frac{1}{4\pi\epsilon_o}.\int_{r_1}^{r_2}\frac{Q}{r^2}dr \ \text{Or}, \ V = \frac{Q}{4\pi\epsilon_o}.\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

And finally the capacitance of a spherical air capacitor in SI system will be $C = \frac{Q}{V} = \frac{4\pi\epsilon_o r_1 r_2}{r_2 - r_1}$

Similarly, this capacitance of spherical air capacitor in cgs system will be $C = \frac{Q}{V} = \frac{r_1 r_2}{r_2 - r_1}$.

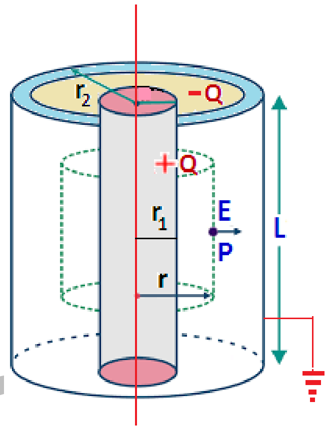


Again for spherical dielectric capacitor, its capacitance will similarly become

$$C = \frac{Q}{V} = \frac{4\pi\epsilon_o k r_1 r_2}{r_2 - r_1}\ [SI] \quad \text{and} \quad C = \frac{Q}{V} = \frac{k r_1 r_2}{r_2 - r_1}\ [cgs]$$

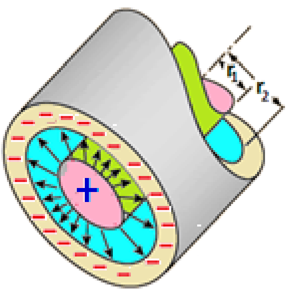
c) Cylindrical Capacitor:

To construct such capacitor we take two concentric cylindrical conducting shells separated either by air or dielectric. So if the outer shell be made grounded with inner shell charged by the charge Q then for their inner and outer radius r_1 and r_2, the electrostatic field at any intermediate point P within two shells of that cylindrical air capacitor at a distance r from the common axis of that capacitor will be



$$E = -\frac{dV}{dr} = \frac{\sigma}{\epsilon_o} = \frac{1}{\epsilon_o} \cdot \frac{Q}{2\pi r l}$$

Or,



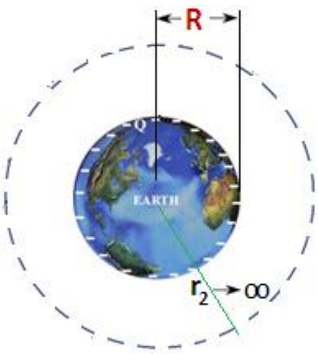
$-\int_V^0 dV = \frac{Q}{2\pi\epsilon_o l} \cdot \int_{r_1}^{r_2} \frac{1}{r}\, dr$ Or, $V = \frac{Q}{2\pi\epsilon_o l} \cdot \ln\left(\frac{r_2}{r_1}\right)$ and finally the capacitance of a cylindrical air capacitor in SI system will be $C = \frac{Q}{V} = \frac{2\pi\epsilon_o l}{\ln\left(\frac{r_2}{r_1}\right)}$. Similarly, this capacitance of cylindrical air capacitor in cgs system will be $C = \frac{Q}{V} = \frac{l}{2\ln\left(\frac{r_2}{r_1}\right)}$. Again for cylindrical dielectric capacitor, its capacitance will be

$$C = \frac{Q}{V} = \frac{2\pi\epsilon_o k l}{\ln\left(\frac{r_2}{r_1}\right)}\ [SI] \quad \text{and} \quad C = \frac{Q}{V} = \frac{kl}{2\ln\left(\frac{r_2}{r_1}\right)}\ [cgs]$$

2. Capacitance of an Isolated Spherical Conductor:

Here we consider a sphere of radius r. This can now be treated as a spherical air capacitor with $r_1 = r$ and $r_2 \to \infty$. So the capacitance of an isolated spherical body in SI system will become $C = \lim_{r_2 \to \infty} \frac{4\pi\epsilon_o r_1 r_2}{r_2 - r_1} = \lim_{r_2 \to \infty} \frac{4\pi\epsilon_o r_1}{1 - \frac{r_1}{r_2}} = 4\pi\epsilon_o r_1 = 4\pi\epsilon_o r$

And thus, this capacitance of a single sphere in cgs system will be $C = r =$ the radius of that sphere. Hence by taking earth to be a sphere of radius R = 6400 km, the capacitance of earth should be



$$C = 4\pi\epsilon_o r = 4\pi\epsilon_o R = \frac{1}{\left(\frac{1}{4\pi\epsilon_o}\right)} R$$

$$= \frac{1}{9 \times 10^9} \times (6400 \times 10^3)\ \text{Farad}$$

$$= \frac{1}{9 \times 10^9} \times (6400 \times 10^3) \times 10^6\ \mu F$$

Finally the capacitance of spherical earth is $C]_{earth} = 711.11\ \mu F$