Liquid Drop Model Approach: Semi Empirical Mass Formula:

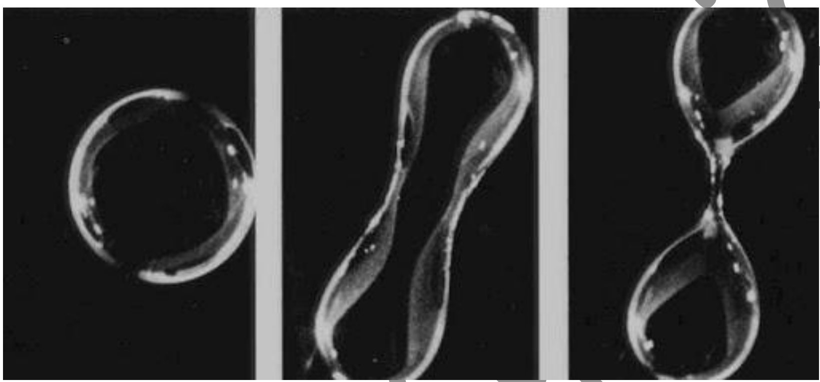
Liquid drop model is one of the famous nuclear models usually associated with the semi-empirical mass formula as given by Bethe – Weizscker and originally the 'Liquid drop model' was suggested by Bohr in 1937. In this model the minute features of nuclear forces are ignored by Bohr but the strong inter nucleon attraction among the nucleons is stressed. The essential assumptions which are made to design such model are:

(a) The nucleus is itself incompressible medium as it consists of incompressible matter

(b) The nuclear force is identical for every nucleon

(c) The nuclear force saturates.

Thus one might expect whether a nucleus can be represented as a crystalline arrangement of nucleons. But it will then assure the zero point vibrations of the nucleons about their mean rest positions and that would be too violent for nuclear stability. Basically the individual nucleons within a nucleus must be able to move like Brownian movement of atoms inside an incompressible liquid drop at any temperature other than absolute temperature and therefore, we should think of a nucleus as being like a small drop of liquid. Such a model is thus known as liquid drop model.



The idea was that the atoms and molecules within a liquid drop correspond to the nucleons in the nucleus is confirmed due to following similarities:

(a) The nuclear forces are analogous to the surface tension force of a liquid;

(b) The nucleons behave in a manner similar to that of molecules in a liquid;

(c) The fact that the density of nuclear matter is almost independent of A shows resemblance to liquid drop where the density of a liquid is independent of the size of the drop i.e. the number of atoms and molecules inside the liquid drop;

(d) The constant binding energy per nucleon is analogous to the latent heat of vaporization;

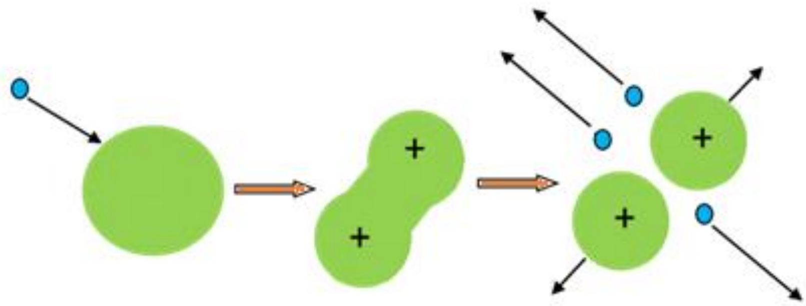
(e) The disintegration of nuclei by the emission of particles is analogous to the vaporisation of molecules from the surface of liquid;

(f) The energy of nuclei corresponds to internal thermal vibrations of drop molecules;

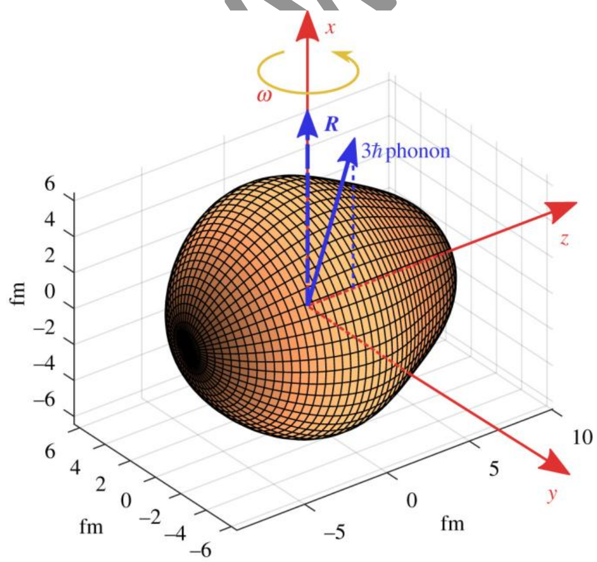
(g) The formation of compound nucleus and absorption of bombarding particles are corresponding to the condensation of drops.

In spite of these similarities, we also see a few differences:

(a) Molecules attract one another at distances larger than the dimensions of the electron shells and repel strongly when the distance is smaller than the size of the electron orbits. Nuclear forces are attractive within the smaller range, the range of nuclear forces



(b) The average K.E. of the molecules within a liquid drop is of the order of 0.1 eV, the corresponding de Broglie wavelength is 5×10^{-11} m which is very much smaller than the inter-molecular distances. The average K.E. of nucleons in nuclei is of the order of 10 MeV, the corresponding $\lambda \simeq 6 \times 10^{-15}$ m, which is of the order of inter-nucleon distances. Hence the motion of the molecules in the liquid can only be explained in classical basis whereas the motion of the nucleons within a nuclear system is of quantum character.



In liquid drop model, since it is taken as a charged liquid drop, deformation of this drop occurs for repulsive Coulomb potential energy associated with the nucleus but such incident doesn't occur in case of neutral liquid drop. Thus if a spherical drop (a nucleus) of radius R_0 is deformed then for $R(\theta, \phi)$ be the distance of the deformed surface from the centre at an angle (θ, ϕ), this can be expressed as

$R(\theta,\phi) = R_0 + \sum_{l=0}^{\infty}\sum_{m=-l}^{l} a_{lm}\ Y_{lm}\ (\theta,\phi)$ where a_{lm} is the concerned coefficient having dimension of length and $Y_{lm}\ (\theta,\phi)$ is spherical harmonics.

While giving binding energy formulation of a nuclear system, two German Scientists Bethe and Weizscker gave Semi-empirical formula for nuclear mass which gives no information about any other properties of nuclei but it only gives the energy information and the Z/A ratio of the nuclear system.

The binding energy of a nucleus is basically given by

$$E_B = \Delta M.c^2 = \{[Zm_p + (A-Z)m_n] - M_n(A,Z)\}c^2$$

But according to Bethe and Weizscker, this nuclear binding energy actually contains five terms and is actually given by $E_B = B = B_1 + B_2 + B_3 + B_4 + B_5$ where

$B_1 = +a_V A = \text{Volume term}$, $B_2 = -a_S A^{2/3} = \text{Surface term}$,
$B_3 = -a_C Z^2/A^{1/3} = \text{Coulomb term}$, $B_4 = -a_A(A-2Z)^2/A = \text{Assymetric term}$,
$B_5 = \pm a_P A^{-3/4} = \text{Pairing term}$.

Thus the expression of nuclear binding energy is

$$E_B = \{[Zm_p + (A-Z)m_n] - M_n(A,Z)\} = a_V A - a_S A^{2/3} - a_C Z^2 A^{-1/3} - a_A \frac{(A-2Z)^2}{A} \pm a_P A^{-3/4}$$

So it is a function of mass no A and hence the nuclear mass is given by

$$M_n(A,Z) = [Zm_p + (A-Z)m_n] - [a_V A - a_S A^{\frac{2}{3}} - a_C Z^2 A^{-\frac{1}{3}} - a_A(A-2Z)^2/A \pm a_P A^{-3/4}]$$

This is semi empirical mass formula for a nuclear system. Let us explain all these terms in nuclear binding energy formulation.

a) Volume Term: Here this term is a positive term in favour of nuclear binding which is the contribution of binding of nucleon within the interior of the nucleus and thus it is proportional to the volume of the nucleus. Thus we have such volume term

$$B_1 \propto V_n \Rightarrow B_1 \propto \frac{4}{3}\pi(R)^3 \Rightarrow B_1 \propto \frac{4}{3}\pi(r_o A^{\frac{1}{3}})^3 \Rightarrow B_1 = +a_V A \leftarrow \text{Volume term}$$

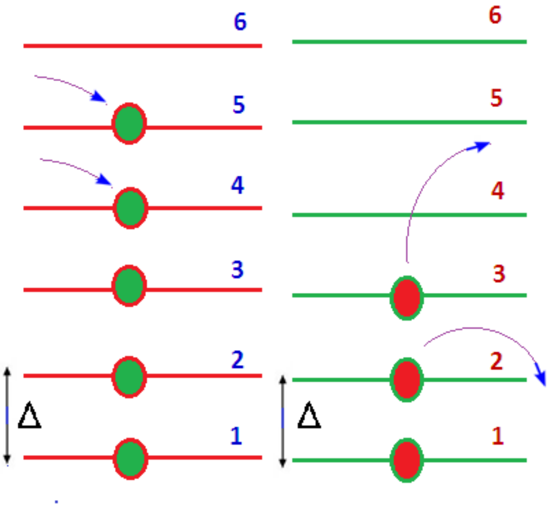
b) Surface Term: Here this term is a negative term against of nuclear binding which is the negative contribution of binding of nucleon on the surface of the nucleus due to surface tension effect and thus it is proportional to the surface area of the nucleus. Thus we have such surface term

$$B_2 \propto S \Rightarrow B_2 \propto 4\pi(R)^2 \Rightarrow B_2 \propto 4\pi(r_o A^{\frac{1}{3}})^2 \Rightarrow B_1 = -a_S A^{2/3} \leftarrow \text{Surface term}$$

c) Coulomb Term: Here this term is a negative term against of nuclear binding which is the negative contribution of binding of nucleon due to repulsive Coulomb self energy stored within the nucleus and thus it is proportional to Coulomb self energy of the nucleus. Thus we have such Coulomb term

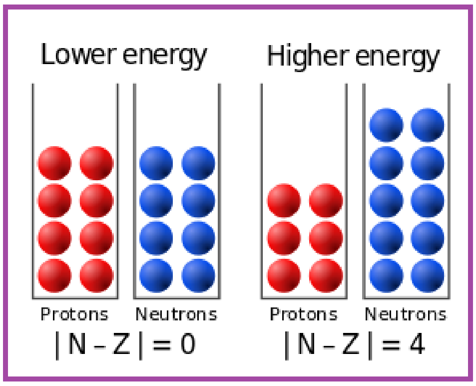
$$B_3 \propto -U_C \Rightarrow B_1 = -\frac{3}{5}.\frac{1}{4\pi\epsilon_o}.\frac{Ze^2}{R} = -\frac{3}{5}.\frac{1}{4\pi\epsilon_o}.\frac{(Ze)^2}{r_o A^{\frac{1}{3}}} = -a_C.\frac{Z^2}{A^{\frac{1}{3}}} \leftarrow \text{Coulomb term}$$

d) Asymmetric Term: Study of no. of neutrons (N) and protons (Z) of stable nuclei shows that for nuclei with $(Z+N)$ up to 18,$(N-Z)=0$ or $+1$, with the exception of hydrogen and $_2He^2$, i.e., there is a sharp tendency for neutrons and protons to pair up. As $(Z+N)$ increases, nuclear forces do not increase much, but repulsion increases with charge. So proton increases in number less rapidly than neutrons, thus heavy nuclei have more neutrons than protons.



Complete theory of this asymmetry term is shown in schematic diagram of nuclear energy levels near the highest filled levels separated by the same energy Δ. Keeping $(N+Z)$ constant, if we remove a proton from level 3 and add a neutron to level 4, $(N-Z)$ becomes 2 and the energy has increased or E_B (BE) has decreased by an amount Δ. Transformation of two nucleons in this way increases the energy by an amount 4Δ (Δ for the first and 3Δ for the second). We thus find as a generalization of this process that the transfer of $\left(\frac{N-Z}{2}\right)$ nucleons decreases the binding energy by an amount $-\frac{\Delta(N-Z)^2}{4}$. Since Δ decreases as 'A' increases, hence decrease in binding energy

$$B_4 = -a_a\frac{(N-Z)^2}{A} = -a_a\frac{(A-2Z)^2}{A}$$
$$\leftarrow \text{Assymetric term}$$



e) Pairing Term: The nuclides with even numbers of protons and neutrons are the most abundant and most stable. Nuclei with odd numbers of both neutrons and protons are the least stable, while nuclei for which either proton or neutron number is odd are intermediate in stability.

To take account of this pairing effect an additional term is used. Conventionally this term is taken as zero for odd $-A$, $-\delta$ for odd-odd and $+\delta$ for even-even nuclides. On the basis of more detailed analysis, we have $\delta = a_p\, A^{-3/4}$ where a_p is the pairing energy constant.

Hence we have $B_5 = 0$ for A odd, Z even N odd or Z odd N even,

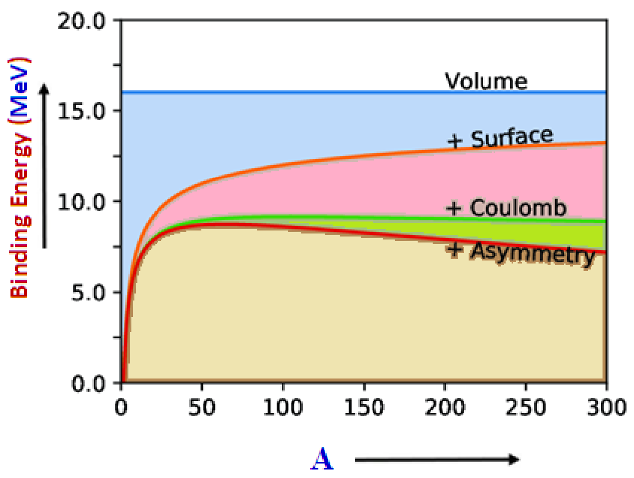
$B_5 = \frac{a_p}{A^{\frac{3}{4}}}$ for A even, Z even and N even, $B_5 = \frac{-a_p}{A^{\frac{3}{4}}}$ for A even, Z odd and N odd

Significance of Several Terms in Binding Energy Formulation:

We have from our previous discussion of Bethe – Weizscker Binding energy formulation that basically in effective binding energy of a nuclear system, there exist at most 5 terms and then it is given by $E_B = B = B_1 + B_2 + B_3 + B_4 + B_5$ where

$B_1 = +a_V A =$ Volume term, $B_2 = -a_S A^{2/3} =$ Surface term,
$B_3 = -a_C Z^2 / A^{1/3} =$ Coulomb term, $B_4 = -a_A(A - 2Z)^2/A =$ Assymetric term,
$B_5 = \pm a_P A^{-3/4} =$ Pairing term.

This formulation gives us the semi empirical mass formula of the nuclear system and it is the overall significance of the whole formulation. As a matter of fact, each binding term has its self importance – either positive or negative in favour or against of nuclear binding and these are



a) Volume term B_1 is in favour of binding and it gives nuclear stability for nucleus having comparatively large volume.

b) Surface term B_2 is a negative term against nuclear binding which gives us 'instability' for nuclear system having comparatively large surface area.

c) Coulomb term B_3 is a negative term against nuclear binding which gives us the departure from the sphericity of nuclear shape and this ensures us the existence of non zero quadrupole moment of a nuclear system.

d) Asymmetric term B_4 is a negative term against nuclear binding which gives us the stability of a nuclear system having equal number of protons and neutrons in absence of this term.

e) Pairing term B_5 is in favour of binding for 'even – even' nucleus and it is against nuclear binding for 'odd – odd' nucleus. But it has no significance for 'odd – even' nucleus.