A few samples of Physics Formula on Class XII Syllabus in +2 Levels:

1. Charge is the homologous parameter of mass which when associated with a body; the body is then called charged body. By the help of charge two so called charged bodies will interact with each other electrostatically. The concept of charge is not yet clear for the macro body but it is taken as a quantum number which is taken as zero for electrically neutral body and non zero for charged body.

Mathematically, this quantum number is given by

$\mathbf{Q = I_z + \frac{1}{2}(B+S)}$ **where I_z = Isospin quantum no, B = Baryon no and S = Strangeness quantum no.**

2. By the theory of charge quantization any amount of positive or negative charge is the integer multiple of a fundamental charge when that fundamental or basis charge is the charge of positron or negatron. Thus for this theory, for any charge

$\mathbf{+Q = \sum_{i=1}^{N}[e^+] = N\,(e+)}$ **and $-Q_o = \sum_{i=1}^{N_o}[e^-] = N_o\,(e^-)$.**

And also $\mathbf{|e^{\pm}| = 1.67 \times 10^{-19}\ Coulomb = 4.8 \times 10^{-10}\ esu\ of\ charge}$

We should note this theory of quantization is only obtained from experimental findings but there is no theoretical background behind it.

Again in quantum mechanics this quantization theory will not be valid for quarks as theoretically it has fractional charge of negatron or positron.

3. Mathematically if q_1 and q_2 be two charges having separation r then magnitude of Coulomb interaction force is $|\vec{F}| = k_o \frac{q_1 q_2}{r^2}$ where k_o is a proportionality constant, not universal and basically it depends on the nature of the medium and the system of unit chosen. And also we observe that

$\mathbf{k_o = 1}$ **(cgs, air ¤) $= \frac{1}{k}$ (cgs, Other medium) $= \frac{1}{4\pi\varepsilon_o}$ (cgs, air) $= \frac{1}{4\pi\varepsilon_o k}$ (SI, Other medium)**

Now by taken SI system and air medium we get the magnitude of Coulomb force

$\mathbf{|\vec{F}| = F = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}}$ **where ϵ_o is electric permittivity of air or vacuum and also $k = \frac{\epsilon}{\epsilon_o} = \epsilon_r$ = Relative permittivity of dielectric constant of the medium and it is 1 for air** or free space. Also we have $\frac{1}{4\,\pi\varepsilon_o} = 9\times10^9\ \mathrm{Nm^2/C^2} = 1\ \mathrm{dyne.cm^2/(esu\ of\ charge)^2}$ where $1\mathrm{C} = 3\times10^9$ esu of charge and $1\mathrm{N} = 10^5\mathrm{dyne}$

Again in any medium other than air the magnitude of Coulomb force is $|\vec{F}| = F = \frac{1}{4\,\pi\varepsilon_o k}\frac{q_1q_2}{r^2}$ and by the rule of vector, we have for air medium in SI system,

$\vec{F} = \frac{1}{4\,\pi\varepsilon_o}\frac{q_1q_2}{r^2}(\pm\hat{r}) = \pm\frac{1}{4\,\pi\varepsilon_o}\frac{q_1q_2}{r^2}(\hat{r}) = \pm\frac{1}{4\,\pi\varepsilon_o}\frac{q_1q_2}{r^3}.\vec{r}$ where +ve and − ve sign respectively indicates the repulsive and attractive force.

4. The dimension of permittivity is given by $[\epsilon] = \frac{[q_1][q_2]}{[F][r^2]} = \frac{AT.AT}{MLT^{-2}L^2} = M^{-1}L^{-3}T^4A^2$

5. If in electrostatic field, a no of discrete charges be present then the effective Coulomb force to a given charge Q_i for all those charges $(q_j;\ j = 1,2,3,4,\ldots..)$ will be the vector sum of the forces between given charge and each of that distribution of discrete charges.

So for N no of discrete charges the effective Coulomb force on that given charge Q_i will be $\vec{F}_i = \sum_{j=1}^{N}\frac{1}{4\pi\epsilon_o}.\frac{Q_iq_j}{r_j^3}\vec{r}_j\ \ (i\neq j)$

6. Electrostatics Field Intensity for a Point Charge is mathematically given by

$\vec{E} = \frac{1}{4\pi\epsilon_o}\frac{Q\times1}{r^3}.\vec{r} = \frac{1}{4\pi\epsilon_o}\frac{Q}{r^3}.\vec{r}$ where the source charge Q is positive.

On the other hand for negative source charge $\vec{E} = -\frac{1}{4\pi\epsilon_o}\frac{Q}{r^3}.\vec{r}$. So in general for any source charge $\vec{E} = \pm\frac{1}{4\pi\epsilon_o}\frac{Q}{r^3}.\vec{r}$.

The unit of this electric field intensity is N/C or $\frac{\mathrm{dyne}}{\mathrm{esu}}$ of chargewhere $1\ \mathrm{N/C} = \frac{10^5\mathrm{dyne}}{3\times10^9\mathrm{esu\ of\ charge}} = \frac{1}{30000}$ dyne/esu of charge

7. On the basis of electrostatics field intensity, the electrostatic field intensity at any field point for the given discrete charge distribution will be $\vec{E} = \sum_{i=1}^{N}\frac{1}{4\pi\epsilon_o}\frac{q_i}{r_i^3}\vec{r}_i$ and similarly for continuous charge distribution, this net field will be $\vec{E} = \frac{1}{4\pi\epsilon_o}\iiint\frac{\rho dV}{r^3}\vec{r}$

8. If φ be the flux passing through cross section A of that electrostatics region then mathematically $|\vec{E}| = \frac{\varphi}{A}$ and $\varphi = \iint\vec{E}.d\vec{s} = \iint\vec{E}.\hat{n}ds$

9. Electric Dipole and its Characteristics:

a) Dipole moment is given by $\vec{p} = q\vec{l}$ and for small dipole, the moment is $d\vec{p} = qd\vec{l}$

b) If a small dipole of dipole moment $\vec{p}$ be placed in external electrostatic field then the electrostatic potential energy stored in it will be $U_e = -\vec{p}.\vec{E}$

c) If an electric dipole be placed in external electrostatic field then the moment of couple acting on it will be $\vec{G} = \vec{p} \times \vec{E}$ and the work done by this couple will be

$$W = -\int_{\theta}^{0} Gd\theta = -\int_{\theta}^{0} pESin\theta d\theta = pE(1 - Cos\theta)$$

d) For a linear dipole the electrostatic field intensity at any end on point at a position x on its axis is given by $\vec{E}]_{\text{end on}} = \frac{1}{4\pi\epsilon_o} \cdot \frac{2px}{[x^2-l^2]^2}\hat{n}$

e) For a linear dipole the electrostatic field intensity at any broad on point on perpendicular bisector of its axis is given by $\vec{E}]_{\text{broad on}} = \frac{1}{4\pi\epsilon_o} \cdot \frac{p}{[x^2+l^2]^{3/2}}\widehat{n_o}$

f) For a small dipole of dipole moment $\vec{p}$, the magnitude of electrostatic field intensity at any field point will be $|\vec{E}| = \frac{1}{4\pi\epsilon_o}\frac{p}{r^3}[3Cos^2\theta + 1]^{1/2}$.

Vectorically, this dipole field at any point due to a small dipole is given by

$$\vec{E} = \frac{1}{4\pi\epsilon_o} \cdot \left(\frac{3(\vec{p}.\vec{r})\vec{r}}{r^5} - \frac{\vec{p}}{r^3}\right)$$

g) For a small dipole for which $r^2 \gg l^2$ the electric field intensity at the axial point for that short dipole will be $E]_{\text{end on}} = \frac{1}{4\pi\epsilon_o} \cdot \frac{2p}{r^3}$.

Similarly, it is at broad on position is given by $E]_{\text{broad on}} = \frac{1}{4\pi\epsilon_o} \cdot \frac{p}{r^3}$

10. Mutual potential energy between two coplanar dipoles is given by

$$U_{21} = U_{12} = \frac{p_1p_2}{4\pi\epsilon_0 r^3}[\cos(\theta_2 - \theta_1) - 3\cos\theta_1\cos\theta_2] = \frac{p_1p_2}{4\pi\epsilon_0 r^3}[\sin\theta_1 \sin\theta_2 - 2\cos\theta_1\cos\theta_2]$$

where we have put $r_{21} = r$. This is the interaction energy of two coplanar dipoles separated by a distance r.

If the dipoles lie along the same line, $\theta_1 = \theta_2 = 0$ and then $U_{21} = U_{12} = -\frac{p_1p_2}{2\pi\epsilon_0 r^3}$

11. Solid angle subtended by the area ds at any position r is $d\omega = \frac{dsCos\theta}{r^2} = \frac{d\vec{s}.\vec{r}}{r^3}$

12. By Gauss's law of Electrostatics, the total electrostatics flux over a closed region will be $\varphi = \iint \vec{E}.d\vec{s} = \frac{1}{\epsilon_0}\sum Q$ =Net charge enclosed by that closed region.

This is because of the fact that mathematically we should have

$$\varphi = \iint \vec{E}.d\vec{s} = \iint \frac{1}{4\pi\epsilon_0}\frac{\sum Q}{r^3}\vec{r}.d\vec{s} = \frac{1}{4\pi\epsilon_0}\sum Q \iint \frac{d\vec{s}.\vec{r}}{r^3} = \frac{1}{4\pi\epsilon_0}\sum Q \iint d\omega$$
$$= \frac{1}{4\pi\epsilon_0}\sum Q.4\pi = \frac{1}{\epsilon_0}\sum Q.$$

This is Gauss's law of electrostatics. Differential form of this Gauss's law is given by $\vec{\nabla}.\vec{E} = \frac{1}{\epsilon_0}\rho$

13. We have differential form of Gauss's law $\vec{\nabla}.\vec{E} = \frac{1}{\epsilon_0}\rho$. It is a basic equation in electrostatics. It relates electric field at a point with the charge density at that point. Since $\vec{E} = -\vec{\nabla}\phi$, Equation $\vec{\nabla}.\vec{E} = \frac{1}{\epsilon_0}\rho$ may be rewritten as

$$\vec{\nabla}.(-\vec{\nabla}\phi) = \frac{1}{\epsilon_0}\rho \quad \text{or} \quad \nabla^2\phi = -\frac{\rho}{\epsilon_0}.$$

This is known as Poisson's equation. When $\rho = 0$, it reduces to $\nabla^2\phi = 0$ this is known as Laplace's equation

14. Electrostatics field intensity at a certain normal distance from infinite linear uniform charge distribution: For linear density of charge λ , it is given by $E = \frac{1}{2\pi\epsilon_0}.\frac{\lambda}{x} = \frac{1}{4\pi\epsilon_0}.\frac{2\lambda}{x}$

15. Electrostatic Field Intensity at a point close to the uniformly charged infinite plane sheet: For surface density of charge , it is given by $E = \frac{\sigma}{2\epsilon_0}$

16. Electrostatic Field Intensity at a point close to a Charged Conducting Surface: For surface density of charge , it is given by $E = \frac{\sigma}{\epsilon_0}$

17. For a charged Spherical Shell of radius 'a', the magnitude of electrostatics field intensity at any point at a distance r from the center of the shell is given by

$$E]_{r>a} = \frac{1}{4\pi\epsilon_0}.\frac{Q}{r^2}\ , E]_{r=a} = \frac{1}{4\pi\epsilon_0}.\frac{Q}{a^2} \text{ and } E]_{r<a} = 0$$

18. For a charged solid sphere of radius 'a', the magnitude of electrostatics field intensity at any point at a distance r from the center of the shell is given by

$$E]_{r>a} = \frac{1}{4\pi\epsilon_0}.\frac{Q}{r^2}\ , E]_{r=a} = \frac{1}{4\pi\epsilon_0}.\frac{Q}{a^2} \text{ and } E]_{r<a} = \frac{1}{4\pi\epsilon_0}.\frac{Qr}{a^3}$$

19. Electrostatic Potential at any field point due to a single source charge is given by $V = \pm\frac{1}{4\pi\epsilon_0}\frac{Q}{r}$ where SI unit of this potential is Joule / Coulomb or Volt and CGS unit will be erg / esu of charge or esu of potential where 1 Volt = 1J / 1C = 10^7 erg / (3 x 10^9 esu of charge) = (1/300) esu of potential Or, 1 esu of potential = 300 Volt

20. The electrostatic field and potential at any distance r from a source charge are respectively $E = \pm\frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$ and $V = \pm\frac{1}{4\pi\epsilon_0}\frac{Q}{r}$ and in this case we can write down

$$\frac{dV}{dr} = \frac{Q}{4\pi\epsilon_0}.\frac{d}{dr}\left(\pm\frac{1}{r}\right) = \frac{1}{4\pi\epsilon_0}.Q.\left(\mp\frac{1}{r^2}\right) = -\left[\pm\frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\right] = -E.$$

And the potential field relation is $E = -\frac{dV}{dr}$ and $V = -\int_\infty^r Edr = \int_\infty^r EdrCos\pi = \int_\infty^r \vec{E}.d\vec{r}$. But the actual vector relation between electrostatics field and potential at any field point is given by $\vec{E} = -\vec{\nabla}(V)$

21. Electrostatic Potential Energy for two Static Charges is $U = W = \int_\infty^r \vec{F}.d\vec{r} = \frac{1}{4\pi\epsilon_0}.\frac{qQ}{r}$

22. Relation between Electrostatic Force and Potential Energy is $E = -\frac{dV}{dr}$ we have $qE = -\frac{d(qV)}{dr}$ or $F = -\frac{dU}{dr}$.

It is the relation between electrostatic force and electrostatic potential energy. Vectorically this relation is actually given by $\vec{F} = -\vec{\nabla}(U)$.

23. In electrostatics field region, if we consider such an imaginary surface such that the electrostatic potential at each and every point on that surface will be the same then that imaginary surface e is called equipotential surface.

Now consider that A and B are two close points on that surface having respective position vectors $\vec{r}$ and $\vec{r} + d\vec{r}$ with respect to an arbitrarily chosen origin O. Since they are positioned on the equipotential surface their potential will be equal for which we have $V_A = V_B$ and for these two close points we should have $dV = 0$

Or $-E\ dr = E\ dr\ Cos\ \pi = \vec{E}.d\vec{r} = 0$. So here we see that the two vectors $\vec{E}$ and $d\vec{r}$ are mutually perpendicular to each other.

So the basic characteristics of the equipotential surface is that the electric field intensity at any point of it is perpendicular to that surface at that point and the work done in bringing any charge along that surface will be zero.

24. For electrostatic potential at any point for Discrete or Continuous Charge Distribution we have from the principle of superposition, the electrostatic potential at any field point for the given discrete charge distribution will be $\mathbf{V = \sum_{i=1}^{N} \frac{1}{4\pi\epsilon_o} . \frac{q_i}{r_i} = \frac{1}{4\pi\epsilon_o} \sum_{i=1}^{N} \frac{q_i}{r_i}}$

Again for the continuous charge distribution this potential will be

$$\mathbf{V = \iiint \frac{1}{4\pi\epsilon_o} . \frac{\rho dV}{r} = \frac{1}{4\pi\epsilon_o} . \iiint \frac{\rho dV}{r}}$$

25. For superposition principle in respect of Electrostatics Potential, the potential at any point is the sum of the potential due to individual charges. So using Equation $\boldsymbol{\phi} = \frac{1}{4\pi\epsilon_0} . \frac{q}{|\vec{r}-\vec{r}\prime|}$ and this principle it is possible to calculate potential due to arbitrary charge distributions.

We can write for potential due volume, surface and line charge distributions as

$$\boldsymbol{\phi}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho(\vec{r})dV}{|\vec{r}-\vec{r}\prime|} \text{ (Volume charge distribution)}$$

$$\boldsymbol{\phi}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_S \frac{\sigma(\vec{r}\prime)dS}{|\vec{r}-\vec{r}\prime|} \text{ (Surface charge distribution)}$$

$$\boldsymbol{\phi}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_l \frac{\lambda(\vec{r}\prime)dl}{|\vec{r}-\vec{r}\prime|} \text{ (Line charge distribution)}$$

26. For any small dipole the potential at a distance r from that dipole will be

$$\mathbf{V(r,\theta) = \frac{1}{4\pi\epsilon_o} . \frac{d\vec{p}.\vec{r}}{r^3}}.$$

At that same point if $\mathbf{E}$ be the magnitude of electric field intensity for that small dipole then for its respective radial and transverse components $\mathbf{E_r}$ and $\mathbf{E_\theta}$ we mathematically have

$$\mathbf{E_r = -\frac{\partial V(r.\theta)}{\partial r} = \frac{1}{4\pi\epsilon_o} . \frac{2dp.Cos\theta}{r^3}} \text{ and } \mathbf{E_\theta = -\frac{1}{r}\frac{\partial V(r.\theta)}{\partial\theta} = \frac{1}{4\pi\epsilon_o} . \frac{dp.Sin\theta}{r^3}}$$

So magnitude of field intensity due to small dipole will be

$$\mathbf{|\vec{E}| = \sqrt{E_r^2 + E_\theta^2} = \frac{1}{4\pi\epsilon_o} . \frac{dp.}{r^3} \sqrt{3Cos^2\theta + 1}}$$

Also it can be shown that the vector form of this field intensity is $\mathbf{\vec{E} = \frac{1}{4\pi\epsilon_o} . \left(\frac{3(d\vec{p}.\vec{r})\vec{r}}{r^5} - \frac{d\vec{p}}{r^3}\right)}$

27. For Electrostatic Field and Potential due to a Uniformly Charged Ring, these are respectively given by $\boldsymbol{\phi} = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda dl}{\sqrt{a^2+x^2}} = \frac{1}{4\pi\epsilon_0} . \frac{Q}{\sqrt{a^2+x^2}}$

And $\vec{E} = -\vec{\nabla}\phi = -\hat{x}\frac{d\phi}{dx} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Qx}{(a^2+x^2)^{3/2}}\hat{x}$

28. For Electrostatic Field and Potential due to a Uniformly Charged Disc the potential is given by

$\phi = \frac{\sigma}{2\epsilon_0}\left[\sqrt{a^2+x^2} - x\right]$ for $x > 0$ and $\frac{\sigma}{2\epsilon_0}\left[\sqrt{a^2+x^2} + x\right]$ for $x < 0$ and the electrostatics field is given by $\vec{E} = -\vec{\nabla}\phi = -\hat{x}\frac{d\phi}{dx} = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{x}{\sqrt{a^2+x^2}}\right]\hat{x}$ for $x > 0$

$$\text{and } \vec{E} = -\frac{\sigma}{2\epsilon_0}\left[1 + \frac{x}{\sqrt{a^2+x^2}}\right]\hat{x} \text{ for } x < 0$$

29. For Electrostatic Field Intensity due to a Uniformly Linear Distribution of Charge, It is given by

$$\vec{E} = \frac{\hat{x}\lambda}{4\pi\epsilon_0}\int_{-\theta_1}^{+\theta_1}\frac{\cos\theta\, d\theta}{x} = \frac{\hat{x}\lambda}{4\pi\epsilon_0 x}.2\sin\theta_1 = \frac{\hat{x}}{4\pi\epsilon_0}.\frac{2\lambda L}{x\sqrt{x^2+L^2}}.$$

If the point P is far away from the line charge then $x \gg L$ and we get approximately $\vec{E} = \frac{\hat{x}}{4\pi\epsilon_0}.\frac{2\lambda L}{x^2} = \frac{\hat{x}}{4\pi\epsilon_0}.\frac{Q}{x^2}$.

For a line charge of infinite extent or for points very close to the line charge $L \gg x$ or $\theta_1 = \pi/2$ and we can write $\vec{E} \approx \frac{\vec{x}}{4\pi\epsilon_0}.\frac{2\lambda}{x} = \frac{\vec{x}\lambda}{2\pi\epsilon_0 x}$.

30. For Electrostatic Potential at several point for a Charged Sphere, these are given by $\phi(r) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$ for $r \geq a$ and $\phi(r) = \frac{Q}{4\pi\epsilon_0}\left[\frac{3}{2} - \frac{r^3}{2a^2}\right]$ for $r \leq a$

31. An important consequence of Gauss's law concerns the equilibrium of a charged particle in an electrostatic field. It is shown that a freely movable charge cannot exist in stable equilibrium in free space under the influence of electrostatic fields alone. This fact is often given the name Earnshaw's theorem.

32. Equipotential surfaces and field lines of a dipole: Equipotential surfaces are everywhere perpendicular to the lines of force. Traces of Equipotential surfaces in the yz –plane can be obtained as follows. The potential at any point (r, θ) is

$$\phi(r, \theta) = \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2}$$

Therefore, for an Equipotential line, $r(\theta) = A\sqrt{\cos\theta}$

where $A = \sqrt{p/4\pi\epsilon_0\phi}$ = constant. This equation gives a family of Equipotential lines, where A is different for different lines.

33. Force on a dipole placed in an electric field is given by $\vec{F} = (\vec{p}\,.\,\vec{\nabla})\vec{E}$. It is a general expression valid for both uniform and non-uniform field.

34. Basically the force on unit area or electrostatic pressure is $\vec{F} = \frac{\sigma^2}{2\epsilon_0}\hat{n}$. Obviously the pressure acts in the outward direction irrespective of the sign of σ.

The pressure can also be expressed in terms of the field given by equation $\vec{E} = \frac{\sigma}{\epsilon_0}\hat{n}$ at the conductor surface $P = \frac{1}{2}\epsilon_0 E^2$

35. Electrostatic Energy of an Assembly of Point Charges: For N point charges the expression for U can also be written as $U = \frac{1}{2}\sum_{i=1}^{N}\sum_{\substack{j=1 \\ j\neq i}}^{N}\frac{1}{4\pi\epsilon_0}\frac{q_i q_j}{r_{ij}}$ where the factor $\frac{1}{2}$ is included to avoid double counting of each pair. Note that the terms with $j = i$ are excluded because it represents self-terms.

The electrostatic energy U can also be written in terms of the electrostatic potential. Thus, $U = \frac{1}{2}\sum_{i=1}^{N} q_i\phi_i$ where, $\phi_i = \sum_{\substack{j=1 \\ j\neq i}}^{N}\frac{1}{4\pi\epsilon_0}\frac{q_j}{r_{ij}}$ is the potential at the location of the ith charge due to all other charges excepting q_i.

36. Electrostatic Energy in Terms of Field Distribution is $U = \frac{1}{2}\int_V \rho(\vec{r})\phi\,(\vec{r})\,dV$. Now using the differential form of Gauss's law, $\vec{\nabla}\,.\,\vec{D} = \rho$, we can write $U = \frac{1}{2}\int_V (\vec{\nabla}\,.\,\vec{D})\,\phi\,dV$

Using the vector identity $\vec{\nabla}\,.\,(\phi\vec{D}) = \vec{\nabla}\phi\,.\,\vec{D} + \phi\,(\vec{\nabla}\,.\,\vec{D})$

We get $U = \frac{1}{2}\oint_S (\phi\vec{D})\,.\,d\vec{S} + \frac{1}{2}\int_V \vec{E}\,.\,\vec{D}\,dV$.

The surface integral in equation $U = \frac{1}{2}\oint_S (\phi\vec{D})\,.\,d\vec{S} + \frac{1}{2}\int_V \vec{E}\,.\,\vec{D}\,dV$ goes down like $1/r$. Thus, as the surface S is expanded to include all of space the surface integral vanishes and we are left with $U = \frac{1}{2}\int_{\text{all space}} \vec{E}\,.\,\vec{D}\,dV$

37. Electrostatic Self-Energy of a Uniformly Charged Sphere is

$U = \frac{4\pi\rho^2}{3\epsilon_0}\int_0^a r^4\,dr = \frac{4\pi\rho^2}{3\epsilon_0}\,.\,\frac{a^5}{5} = \frac{1}{4\pi\epsilon_0}\,.\,\frac{3Q^2}{5a}$ where $Q = \frac{4}{3}\pi a^3\rho$ is the total charge on the sphere.

38. For Classical radius of an electron, suppose we consider the electron as a uniformly charged sphere of radius r_0 containing a total charge $-e$.

The energy required to assemble this sphere of charge is $U = \frac{1}{4\pi\epsilon_0} \cdot \frac{3e^2}{5r_0}$

But we have $\frac{1}{4\pi\epsilon_0} \cdot \frac{3e^2}{5r_0} = mc^2$ **or** $r_0 = \frac{3e^2}{20\pi\epsilon_0 mc^2}$