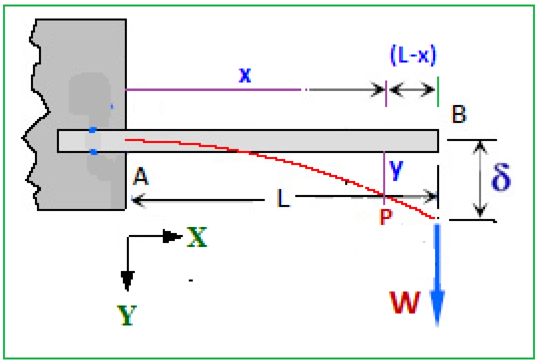
Depression at the Free End of a Light Cantilever for the Loading at that Free End:

Here '**Cantilever**' means a metallic bar of uniform cross section which is made clamped at a rigid support and the other free end is made loaded. Thus the depression of that loaded end will occur for the bending of that bar.

If **G** be the internal bending moment developed within this bent bar for the loading of its free end by the weight **W** then we have in equilibrium for the initial length **L** of this light cantilever,



$$G = \frac{Y}{R}Ak^2 = W(L-x) \text{ ---------------- } (1)$$

Here we have for any point **P** on that bent cantilever, the radius of curvature of this bent cantilever at that point **P** will be

$$R = \frac{(1+y_1^2)^{3/2}}{y_2} \approx \frac{1}{y_2}$$

where $y_1 = \frac{dy}{dx}$ and $y_2 = \frac{d^2y}{dx^2}$ and for small depression δ the slope $y_1 = \frac{dy}{dx} = \tan\theta$ at the point **P** of this bent cantilever is negligible. Thus we have from equation **(1)**

$$YAk^2\frac{d^2y}{dx^2} = W(L-x) \Rightarrow \frac{d^2y}{dx^2} = \frac{W}{YAk^2}(L-x)$$

Integrating both sides we get $\frac{dy}{dx} = \frac{W}{YAk^2}\left(Lx - \frac{x^2}{2}\right) + C_1$

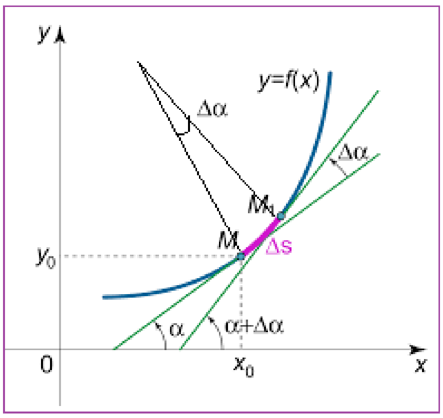
But at $x = 0, \frac{dy}{dx} = 0$ i.e. $C_1 = 0$ and then $\frac{dy}{dx} = \frac{W}{YAk^2}\left(Lx - \frac{x^2}{2}\right)$ Again integrating both sides we get $y = \frac{W}{YAk^2}\left(L\frac{x^2}{2} - \frac{x^3}{6}\right) + C_2$. Also at $x = 0, y = 0$ i.e. $C_2 = 0$

Thus we get $y = \frac{W}{YAk^2}\left(L\frac{x^2}{2} - \frac{x^3}{6}\right)$ and since from figure, at $x = L, y = \delta$ we get the depression at the free loaded end of the cantilever

$$y(x = L) = \delta = \frac{W}{YAk^2}\left(\frac{L^3}{2} - \frac{L^3}{6}\right) = \frac{WL^3}{3YAk^2}$$

This is the depression at the free loaded end of a light cantilever.

[**NB**: The radius of curvature at any portion on a curve is defined by the change of arc length due to unit change of slope angle. Here as shown in figure we



have $R = \frac{ds}{d\alpha}$. We have from figure $\tan\alpha = \text{slope of curve} = \frac{dy}{dx}$

We get $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{d\alpha}(\tan\alpha).\frac{d\alpha}{dx} = \text{Sec}^2\alpha.\frac{d\alpha}{ds}.\frac{ds}{dx}$

So we get $\frac{d^2y}{dx^2} = \text{Sec}^2\alpha.\text{Sec}\alpha.\left(\frac{1}{R}\right)$ and then $\frac{d^2y}{dx^2} = \text{Sec}^3\alpha.\left(\frac{1}{R}\right) = (\text{Sec}^2\alpha)^{\frac{3}{2}}.\left(\frac{1}{R}\right)$

$\frac{d^2y}{dx^2} = (1 + \tan^2\alpha)^{\frac{3}{2}}\left(\frac{1}{R}\right) = \{1 + (\frac{dy}{dx})^2\}^{\frac{3}{2}}.\left(\frac{1}{R}\right)$ Finally we get $R = \frac{(1+y_1^2)^{3/2}}{y_2}$]