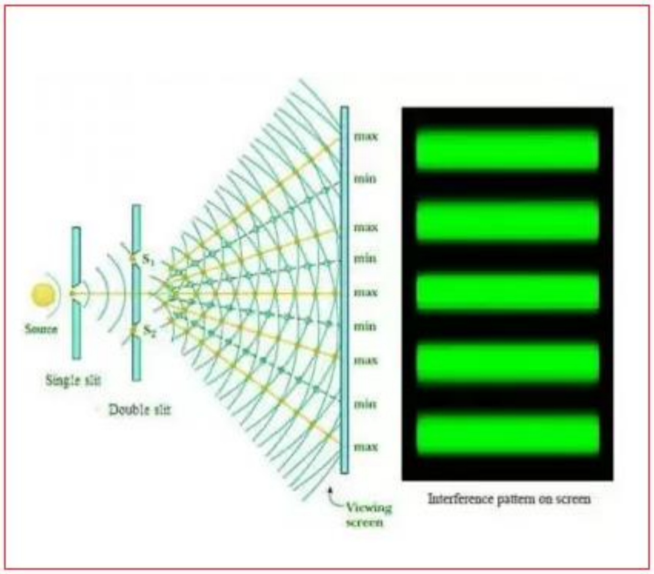
Interference of Light

1. Young's Experiment for Interference of Light:

In the year 1801, the scientist Thomas Young at first observe the property of interference of light. He used two narrow slits lying on the same vertical plane when placed in front of a point source of light. In this arrangement, these two slits will behave like two virtual sources with respect to a screen when placed on the other side of the slits. Here by this optical arrangement, Young showed that alternate bright and dark fringe or band appears on the screen with central fringe in general bright. This is interference of light and the fringes obtained are called interference fringes. For these interference fringes, Huygens observed that



i) The width of every bright or dark fringe will be constant and independent of the position of the fringe

ii) The bright and dark fringes will appear in alternate manner over the screen and the central fringe on the screen will be in general bright

iii) For use of monochromatic light, the color of the bright fringe will be identical to that of the color of the source, but the dark fringes will be totally black

iv) If in any set up of interference the source of monochromatic light be replaced by the source of white light then the central fringe on the screen will be white, the spectrum of white light will appear in each bright fringe and the dark fringes will remain still black

v) If the separation of two sources as used in front of the screen be increased or decreased then the width of the fringe will decrease or increase respectively.

On the other hand if the normal separation between the source and the screen be increased or decreased then the fringe width will increase and decrease respectively. These are the general characteristics of interfere fringes.

2. Conditions of Interference:

To get the stable interference of light, the basic conditions which are required to obeyed are

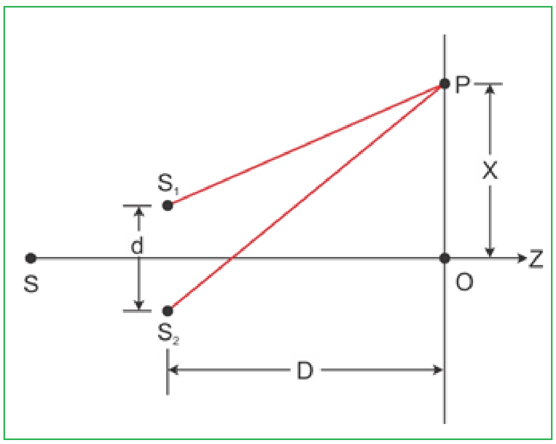
i) The two sources required for interference fringes must be coherent. More clearly the phase difference of two light waves emitted by those two coherent sources will be time independent.

ii) The conditions of all types of coherence will be obeyed.

iii) The amplitude and frequency of two waves coming from two coherent sources will be the same.

iv) The two waves coming from two coherent sources will remain at the same state of polarization at the time of superposition at any screen point. If all these above conditions be obeyed together then the stable interference pattern will be obtained on the screen.

3. Mathematical Conditions of Interference:



Let us now consider that the amplitude and frequency of each of two waves coming from the two coherent sources are A and ω respectively. If φ be their time independent phase difference at the time of superposition at any screen point P then before superposition these two waves can be represented as

$$y_1 = ASin\omega t \text{ and } y_2 = ASin(\omega t + \varphi).$$

Hence the resultant wave for the superposition of those two waves at any screen point will be

$$y = y_1 + y_2 = ASin\omega t + ASin(\omega t + \varphi) = 2ACos\frac{\varphi}{2} Sin\left(\omega t + \frac{\varphi}{2}\right) = A_o Sin\left(\omega t + \frac{\varphi}{2}\right).$$

Here at that screen point P the amplitude of the resultant wave will be $A_o = 2ACos\frac{\varphi}{2}$.

Since the intensity of the wave is directly proportional to the square of the amplitude of that wave, here the resultant intensity of that screen point on superposition will be

$$I_o = kA_o{}^2 = 4kA^2 Cos^2\frac{\varphi}{2}.$$

Now if that screen point P be bright such that the bright fringe or the constructive interference appears at that screen point then

$$I_o = max \text{ and } Cos^2\frac{\varphi}{2} = 1 \Rightarrow Cos\frac{\varphi}{2} = \pm 1 \Rightarrow \frac{\varphi}{2} = n\pi \Rightarrow \varphi = 2n\pi \ (n = 0, 1, 2, 3, \dots\dots\dots).$$

This is the condition of appearance of bright fringe or constructive interference. So in this case the phase difference of two waves on superposition will be even integer multiple of π.

On the other hand if the dark fringe appears at that point $\mathbf{P}$ then we have $\mathbf{I_o = 0}$ and in that case $\frac{\varphi}{2} = 0 \implies \mathrm{Cos}\frac{\varphi}{2} = 0 \implies \frac{\varphi}{2} = (2n+1)\frac{\pi}{2} \implies \varphi = (2n+1)\pi \quad (n = 0, 1, 2, 3, \ldots \ldots \ldots)$

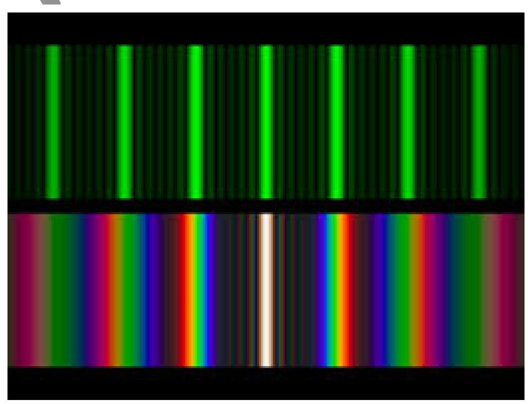
This is the condition of appearance of dark interference fringe or the destructive interference. So in this case the phase difference of two waves on superposition will be odd integer multiple of π. But if the intensity of any point on the screen does not match with any of these two conditions then the intensity of that point will be intermediate, neither maximum nor minimum. These are the mathematical conditions of appearance of interference of light.

4. To find the Position of Interference Fringe and Fringe Width:

Now we consider that the separation of two coherent sources as constructed in front of the screen of an interference set up is $\mathbf{d}$. Again the normal separation between the source and the screen is $\mathbf{D}$. Thus if we take a screen point $\mathbf{P}$ at a distance $\mathbf{x}$ from the central point then for superposition of two waves at that screen point coming from the coherent sources, the path difference of that two waves will be $\Delta = (S_2P - S_1P)$. Now geometrically we can write down $(S_2P)^2 = D^2 + (x + \frac{d}{2})^2$ and $(S_1P)^2 = D^2 + (x - \frac{d}{2})^2$

Hence $(S_2P)^2 - (S_1P)^2 = (S_2P - S_1P)(S_2P + S_1P) = 2xd$

Here we consider for d<<D, $S_2P \approx S_1P \approx D$, then $\Delta.2D = 2xd \implies \Delta = \frac{xd}{D}$.



Again the phase difference of those two waves on superposition at that point will be

$\varphi = \frac{2\pi}{\lambda}(\Delta) = \frac{2\pi}{\lambda}\left(\frac{xd}{D}\right)$. So if interference bright fringe appears at that point then

$$\varphi = \frac{2\pi}{\lambda}(\Delta) = \frac{2\pi}{\lambda}\left(\frac{xd}{D}\right) = 2n\pi \implies x = x_n = \frac{n\lambda D}{d}$$

This is the position of nth bright interference fringe with respect to the central fringe on the screen. On the other hand if dark fringe appears at that point then we have

$$\varphi = \frac{2\pi}{\lambda}(\Delta) = \frac{2\pi}{\lambda}\left(\frac{xd}{D}\right) = (2n+1)\pi \implies x = x_n = \frac{(2n+1)\lambda D}{2d}.$$

This is the position of nth dark interference fringe with respect to the central point on the screen. Hence the fringe width of interference fringe which is defined by the distance between two successive brighter dark fringes is now given by

$$\beta = x_{n+1} - x_n = \frac{(n+1)\lambda D}{d} - \frac{n\lambda D}{d} = \frac{\lambda D}{d} \implies \beta = \frac{\lambda D}{d}.$$

This is fringe width which is constant and independent of position of the fringe.