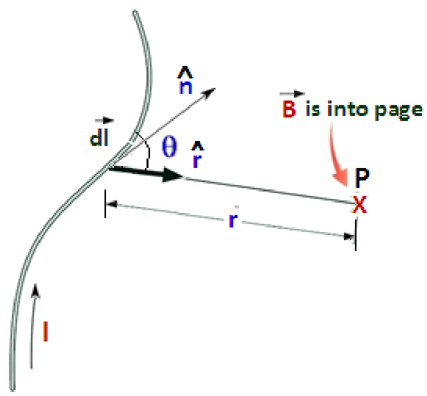
Magnetic Effect of Steady Current

1. Biot – Savart Law or Laplace's Law:

In general a static charge creates electrostatics field at any point where as a moving charge or current flow produces magnetic field or magnetic induction in the neighbor region. The magnitude and direction of the magnetic field or magnetic induction can be determined by this Biot-Savart law or Laplace's law.



By this law, consider a current carrying conductor **AB** carrying current **i** for whom we have to find out the magnetic induction at any neighbor point **P**. Now if the whole conductor be divided into large number small elementary segments then for such a small segment **dl**, if $d\vec{B}$ be the magnetic induction at that point **P** then by this Biot-Savart law or Laplace's law the magnitude of this magnetic induction for that small segment is given by

$$|d\vec{B}| = dB \propto i, \propto dl, \propto \text{Sin}\theta, \propto \frac{1}{r^2} \text{ where } \theta = \text{angle between } \vec{dl} \text{ and } \vec{r}.$$

I.e. $dB \propto \frac{idl\text{Sin}\theta}{r^2}$ and $dB = \frac{\mu_o}{4\pi}\frac{idl\text{Sin}\theta}{r^2}$

Here $\frac{\mu_o}{4\pi}$ is a proportionality constant which has value $\frac{\mu_o}{4\pi} = 10^{-7}\ \text{Tesla}.\frac{\text{m}}{\text{amp}}$

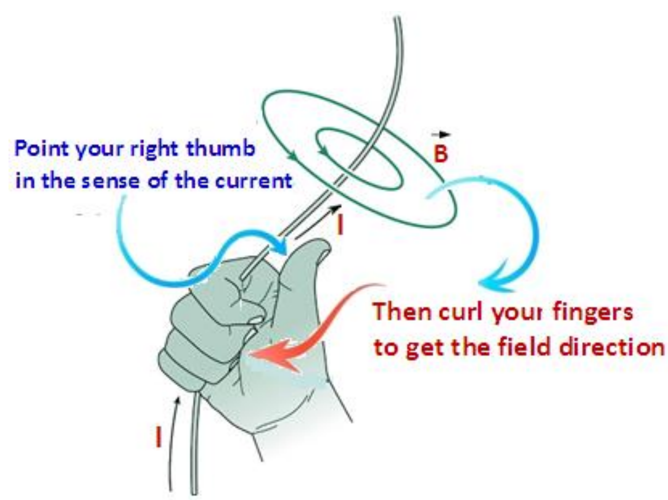
And $\frac{\mu_o}{4\pi} = 1\ \text{Gauss}.\frac{\text{cm}}{\text{emu of Current}}$ where $1\ \text{Tesla} = 10^4\ \text{Gauss}$

and $1\ \text{emu of Current} = 10\ \text{Amp.}$

So for whole current carrying conductor the magnitude of magnetic induction will be

$$|\vec{B}| = B = \int dB = \oint \frac{\mu_o}{4\pi}\frac{idl\text{Sin}\theta}{r^2}$$
$$= \frac{\mu_o}{4\pi}\, i \oint \frac{dl\text{Sin}\theta}{r^2}$$

where the limit of integration will depend on the geometric configuration of the conductor used.



Thus by the rule of vector this magnetic induction will be, since

$$\left|\vec{B}\right| = \frac{\mu_o}{4\pi}\, i \oint \frac{dlSin\theta}{r^2} = \frac{\mu_o}{4\pi}\, i \oint \frac{dlSin\theta}{r^2} = \frac{\mu_o}{4\pi}\, i \oint \frac{dlSin\theta}{r^2},$$

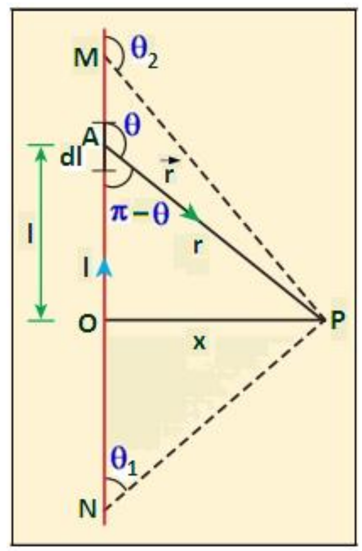
Magnetic induction at that field point will be $\vec{B} = \frac{\mu_o}{4\pi}\, i \oint \frac{(d\vec{l}\times\vec{r})}{r^3}$ **→this is mathematical representation of Biot-Savart law or Laplace's law.**

This integration is called Laplace's Integral which gives both direction and magnitude of magnetic induction at any neighbor point of a current carrying conductor.

On the other hand the magnetic field intensity at that field point will be $\vec{H} = \frac{1}{4\pi}\, i \oint \frac{(d\vec{l}\times\vec{r})}{r^3}$ [since in air or vacuum $\vec{B} = \mu_o \vec{H}$]

2. Application of Biot-Savart Law:

a) To find Magnetic Induction at a certain normal distance from straight finite current carrying conductor:



Here we consider a finite straight current carrying conductor AB, carrying current i. For this current carrying conductor the magnetic induction at a point P at normal distance x from this conductor is now required to determine.

For this purpose we have from Laplace's law, if dB **be the magnitude of magnetic induction at that point P for the small segment** dl **of the conductor then** $dB = \frac{\mu_o}{4\pi}\frac{idlSin\theta}{r^2}$.

Again we have from figure

$\frac{l}{x} = Cot(\pi-\theta) = -Cot\theta$ **Or,** $dl = x.Cosec^2\theta.d\theta$

And $\frac{r}{x} = Cosec(\pi-\theta) = Cosec\theta$ **Or** $r^2 = x^2Cosec^2\theta$.

Thus magnetic induction for the whole conductor

$$\left|\vec{B}\right| = B = \frac{\mu_o}{4\pi}\, i \int_{\theta=\theta_1}^{\theta_2} \frac{x.Cosec^2\theta.d\theta.Sin\theta}{x^2Cosec^2\theta} = \frac{\mu_o}{4\pi}.\frac{i}{x}(Cos\theta_1 - Cos\theta_2).$$

We should note that for very long or infinite straight current carrying conductor, $\theta_1 = 0$ **and** $\theta_2 \to \pi$ **and then in that case the magnitude of magnetic induction at normal distance** x **from that infinite straight current carrying conductor will be**

$$\left|\vec{B}\right| = B = \frac{\mu_o}{4\pi} \times \frac{i}{x}(Cos0 - Cos\pi) = \frac{\mu_o}{4\pi}.\frac{2i}{x}$$

b) To Find Magnetic Induction at an axial point of a Circular Current Carrying Conductor:

Now we consider a circular current carrying conductor having radius a which carries current i. For this conductor, we have to find the magnetic induction at an axial point at a distance x from the center of the conductor.

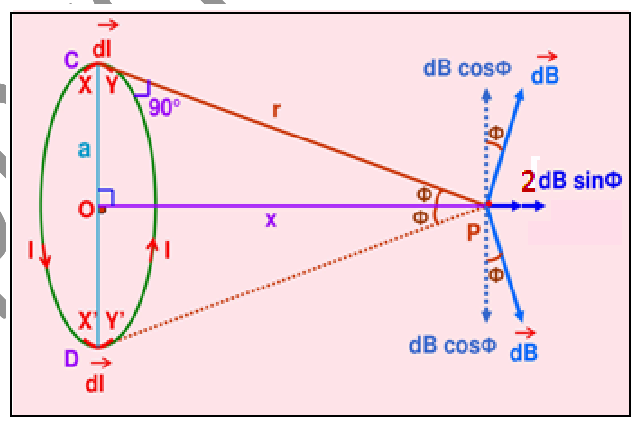
As shown in figure, the effective magnetic induction at that axial point P due to two diametrically opposite segments dl will be $2dBSin\varphi$.

Hence this magnetic induction at P for the whole circular current carrying conductor will be

$$B = \int 2\,dBSin\varphi = 2\int \frac{\mu_o}{4\pi}\frac{idlSin\left(\frac{\pi}{2}\right)}{r^2}.Sin\varphi \;\; (half\ circle)$$

$$= 2\frac{\mu_o}{4\pi}.\frac{i}{r^2}.Sin\varphi.\int_{hc} dl = 2\frac{\mu_o}{4\pi}.\frac{i}{r^2}.\frac{a}{r}.(\pi a) = \frac{\mu_o}{4\pi}.\frac{2\pi i a^2}{(a^2+x^2)^{3/2}}.$$

From this expression we can say that the magnetic induction at the center of the circular current carrying conductor will be $B]_{at\,x=0} = \frac{\mu_o}{4\pi}.\frac{2\pi i}{a}$.



Again if we consider a circular coil of number of turns n then the magnetic induction for that coil will be

$B = \frac{\mu_o}{4\pi}.\frac{2\pi n i a^2}{(a^2+x^2)^{3/2}}$ **and** $B]_{at\,x=0} = \frac{\mu_o}{4\pi}.\frac{2\pi n i}{a}$